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ON THE RADIATION OF MERCURY IN THE MAGNETIC FIELD.[1]

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WE have undertaken in this paper an investigation for the lines of the spectrum of mercury of the connection between the Zeeman effect of the action of a magnetic field upon the light-vibrations and the distribution of the lines in series.[2] This connection was some time ago pointed out by Thomas Preston,[3] but it has not been known to what extent and with what accuracy he demonstrated it. He speaks in his published papers only of the series in the spectra of magnesium, cadmium, and zinc, and in these spectra he has only given the character of the magnetic resolution of the second subordinate series. The investigation of the mercury spectrum by A. A. Michelson[4] refers only to the visible portion in which there is no recurrence of series lines,

[1] Translated from advance proofs from the authors from the Appendix to the *Abhandlungen der K. Akademie der Wissenschaften zu Berlin vom Jahre 1902*. Read at the session on February 6, 1902.

[2] As to series consult the report of RYDBERG, *Rapport au Congrès internat. de Physique*, II, 200, 1900.

[3] *Nature*, **59**, 248, 1899.

[4] ASTROPHYSICAL JOURNAL, **7**, 136, 1898.

and the work of Reese,[1] who studied some of the mercury lines, hardly touches upon the questions we are treating. Kent[2] alone takes up the question, but his results are not, however, in accord with our observations.

For producing the spectrum we employed a large Rowland concave grating of 6.5 meters radius, in a solid mounting. An iron frame of strong U-rails (Fig. 1) rests at *A*, *B*, and *C* on three concrete pillars. The semicircle *AB*, of 6.5 m diameter, constitutes a horizontal table of about 30 cm width on which the slit and camera could be set up at will. The grating is at *C*, and the slit at *A*. Two wooden cameras, each 2 m wide, served for photographing the spectrum. With these placed beside each other, a continuous series of photographic plates, in length four meters, could be exposed, so that one exposure furnished the whole spectrum in several orders. In adjusting the grating we found that the spectrum by no means fell on the "Rowland circle" passing through the slit, the grating and the center of curvature, the deviations amounting to as much as 5 cm. This is explained by Cornu, as is well known, by an increase in the distance between the rulings from one side of the grating to the other.[3] It is often desirable to photograph a line simultaneously in several orders, in order to convince oneself of the reality of faint components, for false side lines often occur from faults of the slit or from inaccurate focusing of the camera. Real components must appear in the different orders at different distances according to their wave-lengths. If they do so appear, their reality is very probable. The fixed mounting of the Row-

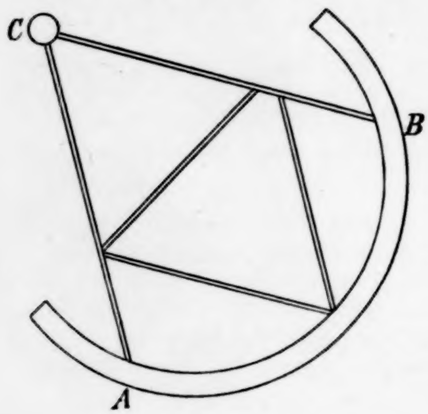

FIG. 1.

[1] ASTROPHYSICAL JOURNAL, 12, 120–135, 1900. [2] *Ibid.*, 13, 289–319, 1901.

[3] See KAYSER'S *Handbuch der Spectroscopie*, I, 441.

land grating has the further advantage that the adjustment does not change, and that greater independence from jars of the building is secured. It was further of importance to us that the magnetic field should be the same for the lines simultaneously photographed. The field-strengths at different exposures may be compared, however, if only one line is common to both exposures. This could be easily arranged in view of the great extent of the region simultaneously photographed. We have largely used this arrangement in the investigation of other elements, and we intend soon to publish the results of this investigation.

For producing the magnetic field we use a Dubois[1] semicircular magnet from Hartmann and Braun, which the Berlin Academy of Sciences was kind enough to place at our disposal.

The source of light is doubtless the most important point in the investigation. We have used Geissler tubes with mercury electrodes in the form suggested by F. Paschen.[2] In the course of the investigation we found it necessary to make several alterations in it. The light which falls upon the grating should come solely from those parts of the source of light which are in the most intense part of the magnetic field. To accomplish this the Geissler tubes were given the form shown in Fig. 2. The capillary crosses the strongest part of the field perpendicularly, and a diaphragm, *AB*, permits only the light from this part to reach the slit. In order to have the same advantage in the case of the ultra-violet rays, a tube closed by a window of fluor-spar was attached at the middle of the capillary (Fig. 3). A quartz window cannot be used on account of the rotary dispersion of quartz, if the polarization of the component is to be determined. Two pieces of equal thickness of right- and left-handed quartz would have to be employed in that case. We placed a calc-spar plate in front of the window of the tubes for investigating the polarization. If an image of the capillary is now projected upon the slit by quartz lenses, it will be separated by the calc-spar into two images, polarized perpendicularly to each other. By a slight change of

[1] *Annalen der Phys.*, 1, 199, 1900. [2] *Physikalische Zeitschrift*, 1, 478, 1900.

the adjusting screws of the lens support we could throw either one of these two images upon the slit. In the correct position of the calc-spar, the one image was composed of light having the electric vibrations in the source of light parallel to the lines of force, while the electric vibrations of the other image are perpendicular to the lines of force. The rotation of the plane of vibration by the passage through the calc-spar by the quartz lenses is without effect upon the result.

The connection between the Zeeman effect and the series is exhibited by the fact that all lines of one series—*i. e.*, all lines

FIG. 2.

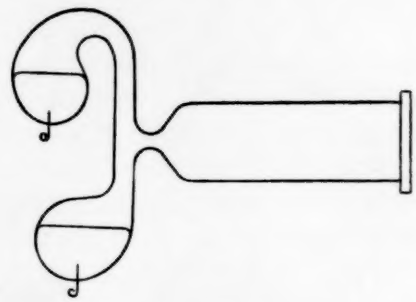

FIG. 3.

whose vibration numbers are expressed by the same formula, when the number of the order runs through the series of whole numbers—are separated by the magnetic field in the same manner, but lines of different series in a different manner. The measurements themselves show best how this is to be understood. Our measures all refer to the same field-strength, although all the exposures were not made at the same field-strength. They can be reduced to the same field-strength, inasmuch as it appears that at different field-strengths the distances of the components of a line remain in proportion, so that the components always give the same image, and only the scale of the image increases with the field-strength. An apparent exception to this rule occurs in case of a few fainter components seen near the strongest lines. It can hardly be doubted, however, that these components do not belong to the lines themselves,

but to satellites which lie very near to them without the magnetic field. If the components of a satellite could be observed alone, it would presumably come out that their distances also remained in proportion when the field-strength changed, although their distances from the component of the principal line did not remain in proportion. A greater dispersion than that of the Rowland grating would be necessary, however, for the satisfactory solution of this question, as the components of the principal lines far too easily cover up the components of the satellite, especially with low field-strengths.

Our observations further show that the increase of the scale of the separation with rising field-strength is always in the same ratio for all lines of the spectrum: if the scale of separation of a line increases in the ratio $a : b$ with a rise of field-strength, then the scale of separation of every other line increases in the same ratio. We have tested this fact for the mercury lines for field-strengths from about 12,000 to about 25,000 c. g. s. units. These observations contradict the statement of N. A. Kent,[1] who, like H. M. Reese, asserts a different behavior for the three zinc lines λ 4680, 4722, and 4811, which show the same separations as the mercury lines at λ 4047, 4359, 5461. According to them, the scale of the separation of λ 5461 does not increase as fast as that of λ 4047 and 4358, when the field-strength rises above 18,000 units. But the objection can be made to the statements of Kent and Reese that they did not obtain the type of the mercury line λ 5461. They speak of it as a diffuse triplet, whereas in fact, as is seen on our plates and as has already been described by Michelson, it consists of nine components, of which the middle three are polarized perpendicularly to the outer six. Kent and Reese, by causing the three interior components to disappear with a nicol, measured the distance of the two exterior groups with their components blended together. Therefore decidedly less weight should be assigned to their measurements than to ours. Whether the scale of the separation increases in direct proportion to the field-strength, or in some other relation,

[1] ASTROPHYSICAL JOURNAL, 13, 294, 1901.

has not been investigated by us, as we have not made any measures of field-strength. Our reduction of all the observations to the same field-strength does not assume direct proportionality, but only that the scale of separation for different lines depends in the same manner on the field-strength.

The details of the reduction are as follows: The differences of wave-length of the nine components of the strong green line λ 5460.97, which is the most widely separated of all the lines of the spectrum by the magnetic field, were measured on five of the best plates. Thirteen unknowns were now determined by the method of least squares, viz., the four factors with which the measures of four of the plates are to be multiplied in order to reduce them to the scale of the fifth, and the nine corrections of wave-length which are to be applied to the measures on the fifth plate. It was at the same time possible to so transform the normal equations that four equations suffice for the four factors. Each of the nine wave-length corrections then comes out as a linear function of the four factors. The other lines of the five plates were now also reduced with the four factors thus obtained, and the separations tabulated below are the means of the values thus found. The field-strengths of the five plates do not differ very greatly from each other. The scales of the separation deviate in the most extreme case by 23 per cent. from each other.

Numerous other plates were, however, also employed for many of the lines, particularly for the fainter ones. In order to reduce these to the same field-strength, the means of the separations of the lines λ 5461, 4359, and 4047, as yielded by the five best plates, were assumed as correct, and for each new plate the reduction factor was determined by the method of least squares, but without the introduction of the wave-length corrections of the separate components as unknowns. Then we have to deal with only one unknown besides the reduction factor sought for, namely, the parallel displacement of the new plate.' The parallel displacement is determined, as easily appears from the method of least squares, by the fact that the

center of gravity of the components for the new plate must agree with the center of gravity of the given components.

We did not directly determine the field-strengths to which all the measurements are reduced, but only obtained it as 24,600 c. g. s. units with the aid of measures by Michelson, Reese, Marchand, and Blythswood. The computation is given at the end of the paper.

SECOND SUBORDINATE SERIES I.

Undisturbed Wave-length	Wave-length in magnetic field[1] Parallel	Wave-length in magnetic field[1] Perpendicular	Mean Error	Intensity	$\Delta\lambda$	$-\Delta\lambda/\lambda^2$	Remarks
		1.605		3	+635	−2.13	On the side toward shorter wave-lengths can be seen two more components, and on the side toward longer wave-lengths, one more, vibrating perpendicular to the lines of force, which probably belong to satellites of the principal line, since their distances from the principal line do not vary with the field-strength in the same ratio as the other distances. Other faint lines are also noticed between those given here.
		1.454		5	+484	−1.62	
		1.289		4[2]	+319	−1.07	
	1.127	.....		3	+157	−0.53	
5460.97	0.970	.....	0.0024	4[3]	0	0	
	0.807	.....		3	−163	+0.55	
		0.654		4[2]	−316	+1.06	
		0.483		5	−487	+1.63	
		0.324		3	−646	+2.17	
		1.929	0.0034	<1	+229	−2.05	
		1.876	0.0024	1	+176	−1.58	
		1.816	0.0024	2	+116	−1.04	
	1.762	.....		1	+ 62	−0.56	
3341.70	1.700	.....	0.0034	2	0	0	
	1.639	.....		1	− 61	+0.55	
		1.584	0.0024	2	−116	+1.04	
		1.524	0.0024	1	−176	+1.58	
		1.462	0.0034	<1	−238	+2.13	
		5.604	0.006	1	+ 94	−1.10	The components are only observed as separate when the other components vibrating parallel to the lines of force are suppressed.
2925.51	5.510	.....	.....	1	......	......	
		5.416	0.006	1	− 94	+1.10	

[1] The first three figures of the wave-length are omitted.

[2] The components 0.654 and 1.289 were often stronger than 0.483 and 1.454; and often the reverse was true.

[3] The intensity of the innermost components is less than that of their neighbors in tubes with the chamber, the innermost being absorbed in the chamber. We also convinced ourselves of this outside of the magnetic field with an echelon spectroscope. If the pressure in the tube is raised by heating the mercury, the reversal of the lines λ 5461 and 4359 is seen in the tubes with a chamber.

SECOND SUBORDINATE SERIES II.

Undisturbed Wave-length	Wave-length in magnetic field		Mean Error	Intensity	Δλ	−Δλ/λ²	Remarks
	Parallel	Perpendicular					
4358.56		8.968	0.0011	5	+408	−2.15	Two fainter components, vibrating perpendicular to the lines of force, at λ 4359.05 and 4358.07, whose distances from the principal line do not vary with the field-strength proportionally to the other distances, presumably belong to satellites of the principal line.
		8.867		4	+307	−1.62	
	8.668	.....		4	+108	−0.57	
	8.458	.....		4	−102	+0.54	
		8.249		4	−311	+1.64	
		8.150		5	−410	+2.16	
2893.67		3.849	0.0030	2	+179	−2.14	Fainter components, vibrating parallel to the lines of force, are also noticed, which probably belong to the satellites.
		3.808	0.0030	2	+138	−1.65	
	3.713	.....	0.0023	1	+ 43	−0.51	
	3.621	.....	0.0023	1	− 49	+0.58	
		3.534	0.0030	2	−136	+1.62	
		3.496	0.0030	2	−174	+2.08	
2576.31		6.419	0.01	..	+109	−1.64	The components are only observed as separate when other components vibrating parallel to the lines of force are suppressed.
	6.31	.....	....	..	.....	.....	
		6.200	0.01	..	−110	+1.66	

SECOND SUBORDINATE SERIES III.

Undisturbed Wave-length	Parallel	Perpendicular	Mean Error	Intensity	Δλ	−Δλ/λ²	Remarks
4046.78		7.136	0.0022	6	+356	−2.17	Three fainter components, vibrating perpendicular to the lines of force, at 7.223, 7.080, and 6.365 presumably belong to satellites, and not to the principal line, as their distances from the principal line do not vary with the field-strength proportionally with the other distances.
	6.780	.....		7	0	0	
		6.423		6	−357	+2.18	
2752 91		3.086	0.007	1	+176	−2.32	Fainter components, vibrating parallel to the lines of force, are also noticed, which also probably belong to the satellites.
		.....	0.010	3	0	0	
	2.910	2.753	0.007	1	−157	+2.07	

FIRST SUBORDINATE SERIES I.

Undisturbed Wave-length	Wave-length in magnetic field		Mean Error	Intensity	$\Delta\lambda$	$-\Delta\lambda/\lambda^2$	Remarks
	Parallel	Perpendicular					
		3.732		1	+272	−2.06	
		3.672		3	+212	−1.61	
		3.610		2	+150	−1.14	The pairs of lines included in the brace merged into one line when both kinds of vibrations were admitted.
	3.582	.....		5	+122	−0.92	
		3.542		2	+ 82	−0.62	
	3.528	.....		2	+ 68	−0.51	
3663.46 Satellite	3.398	.....	0.0019	2	− 62	+0.47	
		3.370		3	− 90	+0.68	
	3.332	.....		5	−128	+0.97	
		3.318		3	−142	+1.08	
		3.249		4	−211	+1.60	
		3.187		1	−273	+2.07	
		3.428	0.0024	2	+378	−2.81	The two components of greatest wave-length were covered by the components of the line λ 3663.46 at this field-strength. They were observed at less field-strength and then reduced to the larger field-strength.
		3.274	0.0024	1	+224	−1.67	
	3.198	.....	0.0031	2	+148	−1.10	
		3.128	0.0038	<1	+ 78	−0.58	
3663.05 Satellite	3.050	.....	0.0024	4	0	0	
		2.977[1]	0.009	<1	− 73	+0.54	
	2.903	.....	0.0031	3	−147	+1.09	
		2.828	0.0024	1	−222	+1.65	
		2.679	0.0024	3	−371	+2.76	
	3.769	3.769		2	+129	−1.41	Not clearly separated.
3023.64 Satellite	3.734	3.734	0.01	2	+ 94	−1.03	
	3.546	3.546		2	− 94	+1.03	Not clearly separated.
	3.500	3.500		2	−140	+1.53	
		1.792	0.004	3	+112	−1.22	
3021.68 Prin. line	1.705	.....	0.010	3	+ 25	−0.27	
	1.655	.....	0.010	2	− 25	+0.27	
		1.569	0.004	3	−111	+1.21	
2803.69	3.69	3.776	0.014	1	+ 86	−1.10	The components are only observed as separate when the vibrations parallel to lines of force are suppressed.
		3.604	0.014	1	− 86	+1.10	

FIRST SUBORDINATE SERIES II.

		2.102	0.0037	1	+152	−1.55	The components given as simultaneously vibrating parallel and perpendicular to the lines of force appear not to coincide precisely. Those vibrating parallel presumably have a somewhat greater, those vibrating perpendicular a somewhat less distance than that given.
		2.058	0.0030	3	+108	−1.10	
	2.007	2.007	0.0030	4	+ 57	−0.58	
3131.95 Satellite	1.950	.....	0.0037	<1	0	0	
	1.884	1.884	0.0030	4	− 66	+0.67	
		1.843	0.0030	3	−107	+1.09	
		1.784	0.0053	1	−166	+1.69	

[1] This component is very faint, decidedly more so than λ 3663.128, and hence is determined with only slight accuracy.

Undisturbed Wave-length	Wave-length in magnetic field Parallel	Perpendicular	Mean Error	Intensity	Δλ	−Δλ/λ²	Remarks
	1.841?	1.841?	0.0026	<1	+181	−1.85	The components marked with a ? were only observed without the calc-spar. Their state of polarization therefore cannot be given.
		1.815	0.0015	3	+155	−1.58	
	1.764	.....	0.0018	3	+104	−1.06	
3131.66 Satellite	.....	1.716	0.0015	3	+ 56	−0.57	
		1.604	0.0015	3	− 56	+0.57	
	1.555	.....	0.0018	3	−105	+1.07	
		1.502	0.0015	3	−158	+1.59	
	1.452?	1.452?	0.0026	<1	−208	+2.12	
	5.936?	5.936?	0.0010	1	+156	−1.60	
		5.897	0.0007	2	+117	−1.20	
		5.858	0.0007	3	+ 78	−0.80	
	5.819	.....	0.0010	1	+ 39	−0.40	
3125.78	5.780?	5.780?	0.0010	1	0	0	
Prin. line	5.744	.....	0.0010	1	− 36	+0.37	
		5.705	0.0007	2	− 75	+0.77	
		5.664	0.0007	3	−116	+1.19	
	5.627	5.627	0.0014	1	−153	+1.57	
		5.346	0.0043	1	+ 56	−0.79	The components were seen separately only when the components vibrating parallel were suppressed.
2655.29	5.29	.....	......	1	...	....	
Satellite		4.234	0.0043	1	− 56	+0.79	
		3.970	0.012	1	+110	−1.56	
	3.932	.....	0.012	1	+ 72	−1.02	
		3.906	0.012	1	+ 46	−0.65	
2653.86 Satellite	.....	3.812	0.012	1	− 48	+0.68	
	3.788	.....	0.012	1	− 72	+1.02	
		3.754	0.012	1	−106	+1.51	
		2.295	0.006	3	+ 75	−1.07	
2652.22	2.220	.....	0.012	3	0	0	
Prin. line		2.145	0.006	2	− 75	+1.07	

FIRST SUBORDINATE SERIES III.

		7.740	0.003	1	+100	−1.14	
2967.64	7.640	.....	0.004	1	0	0	
Satellite		7.541	0.003	2	− 99	+1.12	
	7.423	7.423	0.004	4	+ 53	−0.60	The middle component is broad.
2967.37	7.370	.....	0.004	3	0	0	
	7.318	7.318	0.004	3	− 52	+0.59	
	4.920	4.920	0.001	1	+ 30	−0.47	The components were seen separately only when the components vibrating parallel were suppressed.
2534.89	4.89	.....	.....	.	...	....	
	4.860	4.860	0.001	1	− 30	+0.47	

LINES OF THE SPECTRUM OF MERCURY NOT BELONGING TO THE SERIES.

Undisturbed Wave-length	Δλ	−Δλ/λ²	Mean Error	Intensity	Remarks
	+369	−1.10	0.0067	1	For all the lines included here, except the last, the two outer components vibrate perpendicular to the lines of force, but the inner parallel to the lines of force.
5790.49	0	0	0.0082	3	
	−399	+1.19	0.0067	1	
	+414	−1.24	0.0017	2	Another faint line lies at λ 5789.32, whose separation in the magnetic field could not be observed.
5769.45	0	0	0.0021	5	
	−415	+1.25	0.0017	2	
	+271	−1.12	......	1	
4916.41	0	0	[1]	2	
	−259	+1.07	......	1	
	+206	−1.09	0.0018	2	
4347.65	0	0	0.0020	4	
	−206	+1.09	0.0018	2	
	+246	−1.31	0.0048	1	
4339.47	0	0	0.0064	2	
	−252	+1.34	0.0048	1	
	+156	−0.92	......	1	The wave-length was determined by Eder and Valenta,[2] who include the line among the band-lines of mercury.
4108.2	0	0	......	2	
	−180	+1.07	......	1	
	+268	−1.61	0.0034	3	
4078.05	0	0	0.0043	6	
	−274	+1.65	0.0034	3	
3984.08	—	—	—	—	The line λ 3984.08 (Kayser and Runge) in an undisturbed condition consists of three components: 4.196; 4.121; and 4.054. On switching on the field we only got a diffuse band.
	+141	−0.92	......	2	The wave-lengths are those given by Eder and Valenta.
3906.6	0	0	......	4	
	−144	+0.94	......	2	
	+165	−1.08	......	1	
3902.1	0	0	......	2	
	−159	+1.04	......	1	
	+ 94	−1.16	......	1	The components could be observed as separate only when the components vibrating parallel to the lines of force were suppressed.
2847.85	0	0	......	..	
	− 94	+1.16	......	1	

[1] The lines of this list for which no mean error is given were observed only once. The accuracy is much less for these.

[2] "Ueber die verschiedenen Systeme des Quecksilbers." *Abhandlungen der Wiener Akademie*, 1894.

LINES OF THE SPECTRUM OF MERCURY NOT BELONGING TO THE SERIES.

Continued.

Undisturbed Wave-length	$\Delta\lambda$	$-\Delta\lambda/\lambda^2$	Mean Error	Intensity	Remarks
2536.72	+115 −115	−1.79 +1.79	0.0028 0.0028	10 10	This strong line splits into two components, which both vibrate perpendicular and parallel to the lines of force. Alongside of each of these components, in the direction of greater wave-lengths, each shows a faint component at a distance of 0.090. We should expect this to belong to a line at λ 2536.81, but we have not observed any line besides λ 2536.72 in the undisturbed condition.

All of our measurements are collected in the foregoing table. Kayser and Runge's values are used for the wave-lengths of the lines when not under the action of the magnetic field. The relative accuracy of the components is of course considerably greater than the absolute, which is here of secondary importance. The designations "parallel" and "perpendicular" signify that the electrical vibrations are parallel or perpendicular to the lines of force. The mean error given refers only to the relative precision. The intensities of the components are estimated on a scale on which the greatest intensity is 10 and the least is 1 or < 1. The column headed $\Delta\lambda$ contains the differences of wave-length of the components from that of the unaffected line in thousandths of a tenth-meter. The column headed $-\Delta\lambda/\lambda^2$ gives the differences of the vibration numbers ($1/\lambda =$ number of waves to the centimeter), λ being measured in centimeters. The lines are arranged in the order of series and increasing vibration numbers. The subordinate series are properly six in number, but each three of them which are congruent with each other when plotted on the scale of vibration numbers, should probably be also designated as a subordinate series. In the second subordinate series all three consist of simple lines, but in the first subordinate series each is accompanied by satellites. The three series of the second subordinate series are placed first, then the three series

of the first subordinate series with the satellites, and finally the lines which do not belong to the series.

So far as the accuracy of the measures permits, lines of the same series (*i. e.*, lines which are represented by the same empirical formula of Kayser and Runge or of Rydberg) exhibit the same separation due to the magnetic field, in the sense that according to the scale of vibration-numbers the components of all the series-lines have the same distances, and that corresponding components are also polarized in the same way. But in case of the fainter lines as a rule not all the components were observed, and at the shorter wave-lengths the components crowd so closely together as to be no longer separable. Thus, for instance, in the series to which the lines λ 5461, 3342, and 2926 belong, nine components of the first (aside from the faint components which we ascribe to satellites), nine components of the second, and only three components of the third were observed. It is nevertheless hardly to be doubted that the separation is also the same in case of the third line. It is confirmed by the observed components, while the absence of some of the components is explained by their slight intensities. The precision with which the same vibration-differences repeat themselves corresponds entirely to the precision of the measures. Thus, for instance, we have for λ 5461 and 3342 the following values of $-\Delta\lambda / \lambda^2$:

λ 5461	λ 3342	Differences	Squares
−2.13	−2.05	−0.08	64
−1.62	−1.58	−0.04	16
−1.07	−1.04	−0.03	9
−0.53	−0.56	+0.03	9
0	0	0	..
+0.55	+0.55	0	..
+1.06	+1.04	+0.02	4
+1.63	+1.58	+0.05	25
+2.17	+2.13	+0.04	16
			Sum 143

$$\sqrt{\frac{143}{8}} = 4.2 \ .$$

The mean error of 0.042 for the difference of the values of $\Delta\lambda/\lambda^2$ agrees sufficiently well with the value which may be computed from the mean errors of the wave-lengths of the components of λ 5461 and 3342. The computed mean error of $\Delta\lambda/\lambda^2$ is 0.011 at λ 5461 and 0.035 and 0.041 at λ 3342. Hence we obtain for the differences of the values of $\Delta\lambda/\lambda^2$ mean errors at λ 5461 and 3342 of 0.037 and 0.042, according as the more accurate or less accurate components of λ 3342 are used.

Only three components can be recognized in case of the third series line λ 2926. But the observed values of $\Delta\lambda/\lambda^2$ are also here in agreement with what was to be expected if we assume that only the strongest component appeared.

The repetition of the type is more difficult to observe in case of the first subordinate series than in the case of the second, because the lines are of shorter wave-lengths, so that the components lie nearer together on the sale of wave-lengths. As far as the accuracy of the measures permits, however, the same separation is also here exhibited by lines of the same series, by both principal lines and satellites. Kent asserts that for lines of the same series the separation is not the same, but that $\Delta\lambda/\lambda^2$, for instance, from *Hg* λ 5461 to *Hg* 3342, increases in ratio from 3 to 4.[1] Inasmuch, however, as he did not resolve the separate components of the lines investigated, the contradiction between our measurements is of small significance.

All of the lines not belonging to the series as far as the strong line λ 2536.72 are separated into three components. The differences of the vibration numbers of the components are nearly the same, but they show deviations considerably in excess of the errors of observation. It cannot be doubted, for instance, that the components of λ 5769 give greater vibration-differences than the components of λ 5790, and similarly those of λ 4339 are greater than those of λ 4348.

A survey of all the vibration-differences occurring is given in the following table. Only the strongest of the series lines is here given for each series ; the others would, as remarked above,

[1] ASTROPHYSICAL JOURNAL, **13**, 316, 1901.

TABLE OF THE VIBRATION-DIFFERENCES $-\Delta\lambda/\lambda^2$ OF THE UNAFFECTED LINES AS COMPARED WITH THE COMPONENTS IN THE MAGNETIC FIELD (24,600 UNITS).

λ																	
5461	....	−2.13 s	−1.62 s	....	−1.07 s	....	....	−0.53p	op	+0.55p	....	....	+1 06 s	....	+1.63s	+2.17s	....
4359	....	−2.15 s	−1.62 s	....	..	....	....	−0.57p		+0.54p	....	....	....	....	+1.64s	+2.16s	....
4047	....	−2.17 s	....	....	....	....	....	....	op	....	....	....	....	....	....	+2.18s	....
3663.5	....	−2.06 s	−1.61 s	....	−1.14 s	−0.92p	−0.62 s	−0.51p	op	+0.47p	+0.68s	+0.97p	+1.08 s	....	+1.60s	+2.07s	....
3132.0	....	....	−1.55 s	....	−1.10 s	....	−0.58p & s		op	+0.67p & s		....	+1.09 s	....	+1.69s	....	....
2967.6	....	....	....	....	−1.14 s	....	....	....	op	....	....	....	+1.12 s	....	....	....	....
3663.0	−2.81 s	....	−1.67 s	....	−1.10p	....	....	−0.58 s	op	+0.54 s	....	....	+1.09p	....	+1.65s	....	+2.76 s
3131.7	....	−1.85?	−1.58 s	....	−1.06p	....	....	−0.57 s		+0.57 s	....	....	+1.07p	....	+1.59s	+2.12?	....
2967.4	....	....	....	....	....	....	−0.60p & s		op	+0.59p & s		....	....	....	....	....	....
3655.0	....	−1.98 s	−1.56 s	−1.35p	−1.19 s	−0.81p	−0.76 s	−0.37p		+0.38p	+0.77s	+0.82p	+1.15 s	+1.33p	+1.58s	+1.99s	....
3125.8	....	....	−1.60?	....	−1.20 s	....	−0.80 s	−0.40p	o?	+0.37p	+0.77s	....	+1.19 s	....	+1.57?	...	....
3650	....	....	....	−1.28 s	....	....	....	−0.32p		+0.36p	....	....	....	+1.27 s	....	....	....
4078	....	....	−1.61 s	....	....	....	....	....	op	....	....	....	....	....	+1.65s	....	....
5769	....	....	....	−1.24 s	....	....	....	....	op	....	....	....	....	+1.25 s	....	....	....
4339	....	....	....	−1.31 s	....	....	....	....	op	....	....	....	....	+1.34 s	....	....	....
5790	....	....	....	....	−1.10 s	....	....	....	op	....	....	....	+1.19 s	....	....	....	....
4348	....	....	....	....	−1.09 s	....	...	....	op	....	....	....	+1.09 s	....	....	....	....
4916	....	....	....	....	−1.12 s	....	....	....	op	....	....	....	+1.07 s	....	....	....	....
4108	....	....	....	....	−0.92 s	....	....	....	op	....	....	....	+1.07 s	....	....	....	....
3907	....	....	....	....	−0.92 s	....	....	....	op	....	....	....	+0.94 s	....	....	....	....
3902	....	....	....	....	−1.08 s	....	....	....	op	....	....	....	+1.04 s	....	....	....	....
2848	....	....	....	....	−1.16	....	....	....	op	....	....	....	+1.16 s	....	....	....	....
2537	....	....	−1.79 p & s	....	....	....	....	....	op	....	....	....	....	....	+1.79 p & s	....	....

yield the same vibration-differences with complete observation. The letter s or p next to the number denotes that the electrical vibrations are perpendicular or parallel to the lines of force.

The series lines are given at the beginning of the table, and first the representatives of the three series which are included under the designation of the second subordinate series. Next follow the representatives of the first subordinate series, so arranged that the satellites and principal lines are put together whose vibration-numbers yield the same differences as the three series of the second subordinate series. This arrangement accurately corresponds to Rydberg's laws for the compound triplets.[1] The lines which do not belong to series are given last.

The table clearly shows a connection between the vibration-differences of the different lines. Particular differences repeat themselves so often and so precisely that it can hardly be ascribed to chance. In the three related series the line of greatest wave-length has the most components, that of the shortest wave-length the least. But while in the second subordinate series the components dropping out at shorter wave-lengths are taken from the middle, in the first subordinate series the lines at the sides drop out. Of the eleven lines not belonging to series seven are separated into components with the same vibration-differences. The same differences occur for most of the series lines, except that here still other components are added. The vibration-differences occurring in the second subordinate series are very nearly equidistant, being in the mean: -2.15; -1.62; -1.07; -0.55; 0; $+0.54$; $+1.06$; $+1.64$; $+2.17$. The observed values are very slightly different from the multiples of ± 0.54, viz.: ± 0.54; ± 1.08; ± 1.62; ± 2.16. These vibration-differences are also most frequently represented among the remaining lines. In particular ± 1.08 is the range of the last seven triplets not belonging to the series. A connection is thus shown between these triplets and the series, which is perhaps ultimately to be ascribed to a constant charge of the ions.

We have determined the field-strengths obtaining in the case

[1] See RUNGE and PASCHEN, *Annalen der Phys.*, **5**, 725, 1901.

of our measures from the measurements of Michelson,[1] Reese,[2] and Blythswood and Marchand,[3] on the assumption that the distances of the components are proportional to the field-strength.

These values resulted:

From Michelson - - -	21367	c.g.s. units, from	4 *Hg* lines
Reese - - - -	26330	" " "	3 *Hg* lines
Reese - - - -	25020	" " "	6 *Cd, Zn, Mg* lines[4]
Blythswood and Marchand,	25030	" " "	4 *Hg* lines

As Michelson and Reese did not fully separate the *Hg* lines, we assign weight 1 to the first two values, and 2 to the last two, and obtain as a mean:

Field-strength = 24633 c.g.s. units (mean error = 1000 units).

A more accurate determination of the field-strength would have been desirable, as the mean error of the field-strength is relatively much greater than that of the vibration-differences of the components.

[NOTE. It is, unfortunately, impossible to reproduce here the excellent illustrations prepared to accompany the original article.—EDS.]

[1] ASTROPHYSICAL JOURNAL, **7**, 136, 1898.

[2] *Ibid.*, **12**, 120–135, 1900. [3] *Phil. Mag.*, **40**, 397, 1895.

[4] We photographed the *Cd, Zn* and *Mg* lines in the magnetic field simultaneously with the *Hg* lines, the electrodes of those metals having been amalgamated.

AN IMPROVED METHOD OF CALCULATING THE ORBIT OF A SPECTROSCOPIC BINARY.

By Henry Norris Russell.

The methods of determining the orbit of a spectroscopic binary from its observed velocity-curve may be divided into two general classes:

1. Geometrical, in which the elements of the orbit are determined from the geometrical properties of the curve, especially its maxima and minima.

2. Analytical, in which the observed radial velocity is developed into a trigonometric series, and the elements are found by comparing this series with the corresponding analytical expression for the velocity.

The only method of the second class known to the writer is that of Wilsing.[1] As developed by him, it is available only for orbits of very small eccentricity. The purpose of the present discussion is to extend this method so that it may be generally available.

The theory of the method may be presented as follows: The period U, and the corresponding value μ of the "mean motion," are given at once by the observed velocity-curve. The radial velocity, being then a known periodic function of the time, may be expanded into a Fourier's series of the form

$$\begin{aligned} v = C_0 + C_1 \cos \mu (t - t_0) + C_2 \cos 2\mu (t - t_0) + \cdots \\ + S_1 \sin \mu (t - t_0) + S_2 \sin 2\mu (t - t_0) + \cdots \end{aligned} \tag{1}$$

where t represents the time, and t_0 the initial epoch.

The coefficients of this series may best be obtained as follows:[2] Divide the period into any even number $2n$ of equal parts, beginning at the epoch t_0. Let $v_0, v_1, \text{----} v_{2n-1}$ be the

[1] *A. N.*, **134**, 90, 1893.

[2] Leverrier, *Annales de l'Observatoire de Paris*, Tome I, pp. 109 ff. Wilsing, *loc. cit.*

corresponding values of the velocity (v_0 corresponding to t_0). Then

$$C_0 + C_n = \frac{1}{n}(v_0 + v_2 + \cdots + v_{2n-2})$$

$$C_0 - C_n = \frac{1}{n}(v_1 + v_3 + \cdots + v_{2n-1}) \, .$$

$$C_1 + C_{n-1} = \frac{2}{n}\left(v_0 + v_2 \cos\frac{2\pi}{n} + v_4 \cos\frac{4\pi}{n} + \cdots\right)$$

$$C_1 - C_{n-1} = \frac{2}{n}\left(v_1 \cos\frac{\pi}{n} + v_3 \cos\frac{3\pi}{n} + \cdots\right)$$

$$C_2 + C_{n-2} = \frac{2}{n}\left(v_0 + v_2 \cos\frac{4\pi}{n} + v_4 \cos\frac{8\pi}{n} + \cdots\right)$$

$$C_2 - C_{n-2} = \frac{2}{n}\left(v_1 \cos\frac{2\pi}{n} + v_3 \cos\frac{6\pi}{n} + \cdots\right) .$$

$$S_1 + S_{n-1} = \frac{2}{n}\left(v_1 \sin\frac{\pi}{n} + v_3 \sin\frac{3\pi}{n} + \cdots\right)$$

$$S_1 - S_{n-1} = \frac{2}{n}\left(v_2 \sin\frac{2\pi}{n} + v_4 \sin\frac{4\pi}{n} + \cdots\right)$$

$$S_2 + S_{n-2} = \frac{2}{n}\left(v_1 \sin\frac{2\pi}{n} + v_3 \sin\frac{6\pi}{n} + \cdots\right)$$

$$S_2 - S_{n-2} = \frac{2}{n}\left(v_2 \sin\frac{4\pi}{n} + v_4 \sin\frac{8\pi}{n} + \cdots\right) .$$

The coefficients C_n, C_{n-1}, etc., are not needed in the later work, but their calculation involves very little additional labor, if any, and furnishes a valuable check on the work, as, since the series (1) is convergent, they must be small.

The number of parts into which the period should be divided, in order to obtain sufficiently accurate values of the coefficients, depends upon the rate of convergence of the series (1), which, in turn, depends upon the eccentricity of the orbit. If this is not more than 0.5 (the largest value that has so far been found in a spectroscopic binary), a division into twelve, or at most sixteen parts, will suffice. The larger number may be used when the form of the velocity-curve is conspicuously different from that of a simple sine-curve.

The numerical values of the coefficients C and S being known, we may transform the series (1) into the form

$$v = A_0 + A_1 \cos \{ \mu (t - t_0) + \alpha_1 \} + A_2 \cos \{ \mu (t - t_0) + \alpha_2 \} + \cdots \quad (2)$$

by setting

$$\begin{aligned} A_1 \cos \alpha_1 &= C_1 & A_2 \cos \alpha_2 &= C_2 \\ A_1 \sin \alpha_1 &= - S_1 & A_2 \sin \alpha_2 &= - S_2 \,. \end{aligned} \quad \cdots \quad (3)$$

We have now to find an analytical expression, of the form (2), for the velocity, in terms of the elements. Let

a be the semi-major axis of the orbit;
e the eccentricity,
i the inclination,
ω the longitude of periastron, measured from the descending node.
r the radius vector of the star,
w its true anomaly,
M its mean anomaly,
M_0 the mean anomaly at the epoch t_0,
z the projection of r on the line of sight,
V the radial velocity of the center of mass,
v that of the bright star.

Then we must have

$$v = V + \frac{dz}{dt} \,. \quad (4)$$

Now,

$$\begin{aligned} z &= r \sin (w + \omega) \sin i \\ &= r \cos w \sin i \sin \omega + r \sin w \sin i \cos \omega \,. \end{aligned}$$

$$\begin{aligned} \therefore \quad \frac{dz}{dt} &= \sin i \sin \omega \frac{d}{dt}(r \cos w) \\ &\quad + \sin i \cos \omega \frac{d}{dt}(r \sin w) \,. \end{aligned} \quad (5)$$

But by well-known formulae for elliptic motion

$$\begin{aligned} r \cos w &= \frac{3}{2} ae + a \left(1 - \frac{3}{8} e^2 + \frac{5}{192} e^4 - \cdots \right) \cos M \\ &\quad + \frac{1}{2} ae \left(1 - \frac{2}{3} e^2 + \frac{1}{8} e^4 - \cdots \right) \cos 2M \\ &\quad + \cdots \,. \end{aligned}$$

$$
\begin{aligned}
r \sin w = \quad & a\left(1 - \frac{5}{8}e^2 - \frac{11}{192}e^4 - \cdots\right)\sin M \\
& + \frac{1}{2}ae\left(1 - \frac{5}{6}e^2 + \frac{1}{12}e^4 - \cdots\right)\sin 2M \\
& + \cdots \quad .
\end{aligned}
$$

Differentiating, remembering that $\frac{dM}{dt} = \mu$, substituting in (5), and, for brevity, setting

$$
\left.
\begin{aligned}
1 - \frac{3}{8}e^2 + \frac{5}{192}e^4 - \cdots &= X_1 \\
1 - \frac{5}{8}e^2 - \frac{11}{192}e^4 - \cdots &= Y_1 \\
1 - \frac{2}{3}e^2 + \frac{1}{8}e^4 - \cdots &= X_2 \\
1 - \frac{5}{6}e^2 + \frac{1}{12}e^4 - \cdots &= Y_2 \text{ , etc.,}
\end{aligned}
\right\} \tag{6}
$$

we obtain

$$
\begin{aligned}
\frac{dz}{dt} = & \mu\ a \sin i\ (Y_1 \cos \omega \cos M - X_1 \sin \omega \sin M) \\
& + \mu\, ea \sin i\ (Y_2 \cos \omega \cos 2M - X_2 \sin \omega \sin 2M) \\
& + \cdots \quad .
\end{aligned} \tag{7}
$$

If in this we set

$$
\begin{aligned}
X_1 \sin \omega = b_1 \sin \beta_1 \ , \quad & X_2 \sin \omega = b_2 \sin \beta_2 \\
Y_1 \cos \omega = b_1 \cos \beta_1 \ , \quad & Y_2 \cos \omega = b_2 \cos \beta_2 \text{ , etc.,}
\end{aligned} \tag{8}
$$

we have

$$
\begin{aligned}
\frac{dz}{dt} = & b_1 \mu\ a \sin i \cos (\ M + \beta_1) \\
& + b_2 \mu\, ea \sin i \cos (2M + \beta_2) \\
& + \cdots \quad .
\end{aligned}
$$

Substituting in (4), and remembering that $M = M_0 + \mu\,(t - t_0)$, we obtain

$$
\begin{aligned}
v = V + & \mu\ a \sin i \cdot b_1 \cos\ \{\mu\,(t - t_0) + \ M_0 + \beta_1\} \\
& + \mu\, ea \sin i \cdot b_2 \cos \{2\mu\,(t - t_0) + 2M_0 + \beta_2\} \\
& + \cdots \quad .
\end{aligned} \tag{9}
$$

This is our desired expression for the velocity in terms of the elements and of the time.

The series (2) and (9), considered as functions of the time, are of the same form. If they are to represent the same quan-

tity, their corresponding coefficients must be equal. That is, we must have

$$
\begin{aligned}
&V = A_0 \,, \\
&b_1 \mu \; a \sin i = A_1 \,, \qquad M_0 + \beta_1 = \alpha_1 \\
&b_2 \mu ea \sin i = A_2 \,, \qquad 2M_0 + \beta_2 = \alpha_2 \,, \text{ etc.}
\end{aligned}
\tag{10}
$$

The first of these equations gives V at once. The other four may be used to determine the other elements by a process of approximation. If in (6) we neglect the terms involving e, we have $X_1 = Y_1 = X_2 = Y_2 = 1$, whence, from (8), $b_1 = b_2 = 1$, $\beta_1 = \beta_2 = w$.

10 The equations (~~9~~) become

$$
\begin{aligned}
&\mu \; a \sin i = A_1 \,, \qquad M_0 + \omega = \alpha_1 \\
&\mu ea \sin i = A_2 \,, \qquad 2M_0 + \omega = \alpha_2 \,,
\end{aligned}
$$

whence

$$
\begin{aligned}
&a \sin i = \frac{A_1}{\mu} \,, \qquad M_0 = \alpha_2 - \alpha_1 \\
&e = \frac{A_2}{A_1} \,, \qquad \omega = 2\alpha_1 - \alpha_2 \,.
\end{aligned}
\tag{11}
$$

These are the final equations of Wilsing's method. In fact, he has limited all his developments to the first term of the series in e, so that the quantities X, Y, b and β do not appear in his discussion.

It is clear that the equations (11) give accurate values of the elements only when e is very small. But, in any case, they give approximate values of e and ω, which, introduced into (6) and (8), give values of b and β, from which, by the aid of (10), much more accurate values of the elements may be deduced.

Limiting ourselves to terms of the second degree in e, we find from (6) and (8):

$$
\begin{aligned}
b_1 &= 1 - e^2\left(\frac{1}{2} + \frac{1}{8}\cos 2\omega\right) \\
b_2 &= 1 - e^2\left(\frac{3}{4} + \frac{1}{12}\cos 2\omega\right) \\
\beta_1 - \omega &= \frac{1}{8}\, e^2 \sin 2\omega \\
\beta_2 - \omega &= \frac{1}{12}\, e^2 \sin 2\omega \,.
\end{aligned}
$$

If we substitute these values in (10) and represent the approximate values of the elements given by (11) by the letters a', e', ω, we obtain

$$\left.\begin{aligned} a'' &= a' \left\{ 1 + \frac{1}{2} e'^2 \left(1 + \frac{1}{4} \cos 2\omega' \right) \right\} \\ e'' &= e' \quad + \frac{1}{4} e'^3 \left(1 - \frac{1}{6} \cos 2\omega' \right) \\ \omega'' &= \omega' \quad - \frac{1}{6} e'^2 \sin 2\omega' \\ M''_0 &= M'_0 \quad + \frac{1}{24} e'^2 \sin 2\omega' \ , \end{aligned}\right\} \qquad (12)$$

where a'', e'', etc., are our new and closer approximations to the true values of the elements. The angles in the above formulæ are of course to be expressed in circular measure.

The terms involving e^4 can be similarly calculated. Computation shows that the largest one — which occurs in the expression for a'' — is of the order of $\frac{1}{3} e^4$ with reference to the principal term. If e' is less than 0.30, this term will affect the calculated value of a by less than one-third of 1 per cent.— an amount which, in comparison with the ordinary errors of observation, may be neglected.

If, then, e' is less than 0.30, the equations (12) are sufficiently approximate.

If e is greater than 0.30, it is most convenient to compute b_1, b_2, β_1, and β_2 by (6) and (8), using the values of e and ω given by (12), and then to apply the following equations (which may easily be derived from (10)):

$$\left.\begin{aligned} a \sin i &= \frac{A_1}{b_1 \mu} \ , \quad M_0 = \alpha_2 - \alpha_1 + \beta_1 - \beta_2 \ , \\ e &= \frac{A_2 b_1}{A_1 b_2} \ , \quad \omega = 2\alpha_1 - \alpha_2 - 2(\beta_1 - \omega'') + (\beta_2 - \omega'') \ . \end{aligned}\right\} \qquad (13)$$

These equations give values of the elements which are within 1 per cent. of the truth, even if $e = 0.70$. If, however, the values of e and ω given by (13) differ much from those taken from (12) for use in the reckoning, it is well to recompute the quantities b and β, using the more accurate values of the elements,

and to solve (13) afresh. But in such a case, and generally when the eccentricity is much greater than 0.4, the method here developed becomes laborious, and the geometrical methods are preferable.

When the final elements have been obtained, the velocity-series may be determined from them, and its agreement with the series derived from observation used to check the computations. In deriving this series, the preceding formulæ can be modified to advantage.

If we set $\frac{1}{2}(Y_1+X_1)=h_1$, $\frac{1}{2}(Y_1-X_1)=k_1$, etc., (7) becomes

$$\begin{aligned}\frac{dz}{dt} &= \mu\, a \sin i\,\{h_1 \cos(M+\omega)+k_1\cos(M-\omega)\}\\ &+\mu e a \sin i\,\{h_2\cos(2M+\omega)+k_2\cos(2M-\omega)\}\\ &+\cdots\,.\end{aligned}$$

The resulting expression for the velocity, including all terms which are greater than $\frac{1}{400}$ of the total range of velocity, provided e is less than 0.4, is as follows, where $c=\mu a \sin i$ and $\theta=\mu(t-t_0)$

$$\begin{aligned} v = {} & V\\ &+ c\left(1-\frac{1}{2}e^2\right)\cos(\theta+M_0+\omega)\\ &- \frac{1}{8}ce^2 \cos(\theta+M_0-\omega)\\ &+ ce\left(1-\frac{3}{4}e^2\right)\cos(2\theta+2M_0+\omega)\\ &- \frac{1}{12}ce^3\cos(2\theta+2M_0-\omega)\\ &+ \frac{9}{8}ce^2\left(1-e^2\right)\cos(3\theta+3M_0+\omega) \qquad (14)\\ &+ \frac{4}{3}ce^3\left(1-\frac{4}{5}e^2\right)\cos(4\theta+4M_0+\omega)\\ &+ 1.627\,ce^4\left(1-\frac{3}{2}e^2\right)\cos(5\theta+5M_0+\omega)\\ &+ 2.025\,ce^5\left(1-\frac{7}{4}e^2\right)\cos(6\theta+6M_0+\omega)\\ &+ 2.56\,ce^6\left(1-2e^2\right)\cos(7\theta+7M_0+\omega)\,.\end{aligned}$$

Should the differential coefficients of v with respect to the elements be required, they may be obtained at once by differentiating this expression.

The foregoing method appears upon trial to be somewhat less expeditious in practice than that of Lehmann-Filhès,[1] which is in general use. It should, however, be somewhat more accurate, as the elements are deduced from twelve or more points of the velocity curve, instead of four.

There are some cases, however, where the geometrical methods — depending as they do upon the maxima and minima of velocity — are inapplicable, and in these cases the present method may be found useful.

One such case would occur when a star had a period of about a year, and the maximum or minimum velocity fell in the interval when it was near the Sun. This phase being unobservable, we should be obliged, by the geometrical method, to bridge the gap by a conjectural curve, which might be seriously in error.

In applying the new method, some of the velocities v_0, v_1, etc., would be unknown. But the conditions that the coefficients C_n, C_{n-1}, etc., must be zero (or, at least, very small), give us a set of equations in which the unknown velocities appear, and from which their approximate value may be found, and approximate elements obtained. With these elements, C_n, etc., may be calculated, and accurate values of the unknown velocities obtained, from which satisfactory elements can be computed.

Another case in which the new method is useful is that of a star attended by two dark companions with commensurable periods. In this case the resultant velocity-curve may have several unequal maxima, and the geometrical methods fail altogether. The analytical method, however, enables us to separate the resultant motion into the two component orbital motions, except when the perturbations are large.

If one companion makes m revolutions, and the other n, in a given period U, the radial velocity of the bright star relative to the center of mass (barring secular perturbations) will be a

[1] *A. N.*, **136**, 17, 1894.

purely periodic function of the time with period U, and may be expanded into a series of the form (1). But the component of this velocity due to the first companion will (barring perturbations) be of the form

$$v_1 = C_m \cos m\theta + C_{2m} \cos 2m\theta + \cdots$$
$$+ S_m \sin m\theta + S_{2m} \sin 2m\theta + \cdots ,$$

where $\theta = \mu (t - t_0)$ as in (14), while that due to the second companion will be of the form

$$v_2 = C_n \cos n\theta + C_{2n} \cos 2n\theta + \cdots$$
$$+ S_n \sin n\theta + S_{2n} \sin 2n\theta + \cdots .$$

If the motion of the bright star is really due to the action of two dark companions which are not large enough to disturb one another's motions much, the velocity of the bright star will be approximately $v_1 + v_2$, and those terms in the series (1) which do not occur in either v_1 or v_2 will have coefficients that are zero, or very small. The two series can therefore be picked out from the series (1), if present, by inspection, so that the method here described enables us to determine whether any periodic motion in the line of sight can be represented by the action of two dark companions which do not disturb one another's motions much, and, if so, what their respective orbits are.

ζ *Geminorum*, whose velocity-curve cannot be represented by motion in a single elliptic orbit,[1] may be a system of this sort.

The method above described suggested itself to the writer during an investigation of this star's motion, which is not yet completed, as the stability of the resulting orbits requires investigation. It is hoped that definite results may be reached before long.

PRINCETON, N. J.
February 14, 1902.

[1] CAMPBELL, ASTROPHYSICAL JOURNAL, 13, 94, 1901.

MEASURES OF ABSOLUTE WAVE-LENGTHS IN THE SOLAR SPECTRUM AND IN THE SPECTRUM OF IRON.[1]

By C. Fabry and A. Perot.

IV. IRON LINES.

The spectrum of iron is one of those most commonly employed as a comparison spectrum in spectroscopic measurements made by interpolation, on account of the brightness and the number of lines in this spectrum and the ease with which it is obtained; the iron lines almost always appear as impurity lines when the electric arc is produced between carbon poles. Many of these lines are sharp enough to serve as good standards. It was therefore desirable to measure with the highest possible precision the wave-lengths of some of these lines.

The source employed is the electric arc taken between two iron rods 1 cm in diameter, held in a simple hand regulator. The current is produced by a battery of storage cells giving 120 volts. The intensity of the current can be regulated by means of a rheostat, and is ordinarily about 8 amperes.

As the spectrum of this source contains an immense number of lines, it is necessary to isolate successively the various lines which are to be measured by means of an instrument of high dispersion. After various experiments with prisms we have adopted a plane grating by Rowland 8 cm wide, 5 cm high, and with 558 lines to the millimeter, used with a single lens which serves both as a collimator and as a projecting lens.

An image of the arc S (Fig. 3) is projected, by means of the lens L, upon the slit F. The light which passes through this slit is thrown by the total reflection prism P upon the lens L' ($f=$ 70 cm) and the plane grating R. The diffracted light returns through L' and forms a real image of the spectrum upon the the second slit F', which permits the passage of only the line

[1] Concluded from p. 96.

which is to be measured. The grating stands on the table of a goniometer; by rotating it the spectrum, which remains in focus, can be made to pass over the second slit. This motion may be produced from a distance.

In order to identify the line which it is desired to measure, it is necessary to see a certain extent of the spectrum. For this purpose the slit F', which is mounted on the carriage of an

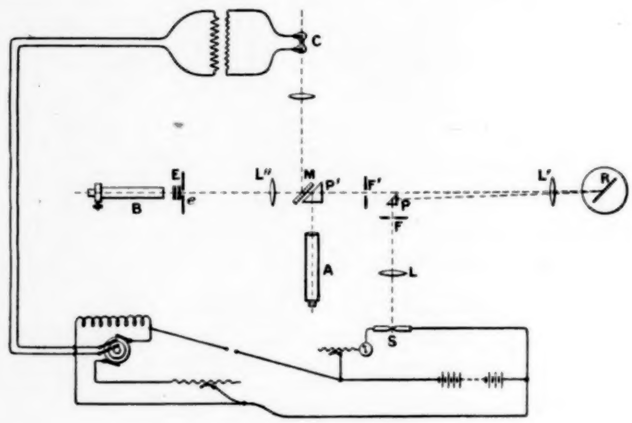

Fig. 3.

optical bench, may be removed, and the spectrum examined by means of a telescope A, used in connection with a total reflection prism P' supported on a platform which can be moved into or out of position by pulling a cord. The cross-wire in the eyepiece of the telescope A is set once for all on the slit F'. To bring a line upon F' the slit is removed, thus permitting a certain extent of the spectrum to be seen, so that the lines can be recognized. The one desired is then set on the cross-hairs and the slit is replaced. In order to identify the lines, we have employed the admirable photographic chart of Kayser and Runge.[1] The interference apparatus (5 mm or 10 mm standard) is placed at E; it is mounted as described above; e is the screen containing a hole 3 mm in diameter through which the light

[1] KAYSER and RUNGE, *Abhandlungen d. K. Akad. d. Wiss. zu Berlin*, 1888.

passes. An image of F' is projected by means of the lens L'' upon this small hole. The image of a cadmium tube C may be projected upon the same diaphragm by inserting the mirror M, which is supported on the same platform with the prism P', with which it moves. The rings are observed and measured with the telescope B focused for parallel rays.

One of the observers measures the diameter of the rings (telescope B), while the other manages the arc and the settings on the spectrum (telescope A); the rod which is used to rotate the grating is within easy reach of his hand. When the platform P' is lowered, observer A can examine the spectrum while B sees the cadmium light; when the carriage is replaced observer B sees the line separated out by F'.

As approximate values of λ' we have used either the values of Kayser and Runge or those of Rowland in the solar spectrum, after having multiplied them by suitable factors which are known approximately, in order to reduce their scale to ours (0.99998 for Kayser and Runge, 0.999963 for Rowland); there is never the slightest doubt about the whole number P'.

The following is an example of the measurement of a line:

Kayser and Runge - - - - - - -	561.581
Rowland ⊙ - - - - - - - - -	561.5877
λ' adopted in computing P' - - - - - -	561.568

	April 2, 1901 5 mm standard		April 16, 1901 10 mm standard	
Observers	Fabry	Perot	Fabry	Perot
P	19484	id.	39462	id.
δ	7.06	7.10	5.79	5.87
$B\delta^2 \times 10^6$	50.18	50.75	33.64	34.59
P'	17646	id.	35739	id.
δ'	5.72	5.80	5.21	5.32
$B'\delta'^2 \times 10^6$	32.83	33.75	27.20	28.36
$\frac{P\lambda}{P'}$	561.55613	id.	561.56238	id.
λ'	561.5659	561.5657	561.5660	561.5659
Corr. for phase	—0.0002	—0.0002	—0.0001	—0.0001
λ' corrected	561.5657	561.5655	561.5659	561.5658

λ' mean - - - - - - - - -	561.5657
Maximum relative error - - - - -	3.6×10^{-7}

The 5 mm standard has been used for all the lines measured; only a few are sufficiently fine to permit the use of the 10 mm standard. Finer lines might have been obtained by means of an arc in vacuo;[1] the wave-lengths would have been more accurately determined, but they would have differed slightly from those derived from the arc in air, as used by spectroscopists.

There is no occasion to compare our results with those of other observers: those of Kayser and Runge are not sufficiently accurate, while Rowland's refer in almost all cases to the Sun, and it is well known that there may be a considerable difference between the wave-lengths of the same line in the Sun and in the arc. Our results are given in Table II at the end of this article.

V. SOLAR SPECTRUM.

The innumerable lines of the solar spectrum furnish the most complete spectroscopic scale imaginable; it is for this reason that spectroscopists have always chosen their fundamental standards from this spectrum. In particular, the application of Rowland's method of coincidences, which requires numerous standards, would doubtless have been impossible without this collection of lines. In addition to its very great intrinsic interest, the solar spectrum is of general importance.

Since the time of Rowland's investigations, all spectroscopic measures have been interpolated by means of his wave-lengths, and consequently refer to his scale. Is this scale perfectly correct? Are its relative wave-lengths perfectly exact? Furthermore, what is the precise unit adopted? It was in the hope of answering these questions that we undertook the following investigations. It might have sufficed to measure the wave-lengths of a certain number of bright lines, which could have been referred to Rowland's scale by interpolation with respect to the solar lines. It seemed more certain and at the same time more elegant to compare directly, by an interference method, the wave-lengths of a certain number of solar lines with the green line of cadmium. Fortunately the special properties of

[1] *Journal de Physique* (3), **9**, 369, 1900.

interference phenomena with silvered films rendered this direct interference measurement possible.

The method adopted was as follows: Project a real image of the solar spectrum upon a slit, F', and illuminate the interference apparatus with the light which passes through it. This light, whose homogeneity increases with the fineness of the slit and the purity of the spectrum, will give a system of interference rings, provided the difference of path is not too great. If the slit F' is gradually widened, the light becomes less and less homogeneous, and the fringes become confused and soon disappear. Let us now suppose that in the region of the spectrum occupied by the slit F' there is a solar line of wave-length λ'. In the complex light transmitted by the slit, this radiation is lacking; there will thus be in the field of the telescope a system of dark fringes, having precisely the same properties as the bright fringes given by a line of wave-length λ'. The method of measuring λ', based on a determination of diameters, is thus applicable to this system of rings. It is evident that the slit must not be too wide, in order that the dark fringes may not be lost in a uniform illumination. The same effect results from widening the slit which produces the spectrum. In fact, it is advantageous to widen both slits, and maximum brightness corresponds to the case where the two slits are equal.

Arrangement of the apparatus.— It is necessary that the spectrum shall be sufficiently dispersed to separate out with certainty the line which is to be measured, and this, in certain parts of the spectrum, requires very great dispersion. We have employed the second order spectrum given by a concave Rowland grating of the largest size (14.5 cm by 5 cm ruled surface, 6.50 m radius of curvature, 568 lines to the millimeter). This grating is mounted in a manner different from that ordinarily employed; in Rowland's classic mounting the illuminating slit is fixed, and the diffracted image changes its position in passing from one line to another. It would have been necessary to render movable the slit F', and with it the interference apparatus, the observing

telescope, and the cadmium tube. The preparations required for each measurement would have taken a long time, and we therefore preferred to fix in position all parts of the apparatus employed in the interference work. For this purpose it is necessary that the slit F' be fixed and that the light which traverses it also be fixed in direction, and hence that the grating shall not be moved. In passing from one line to another it is therefore necessary to displace the illuminating slit F without cutting off from it the light of the solar image.

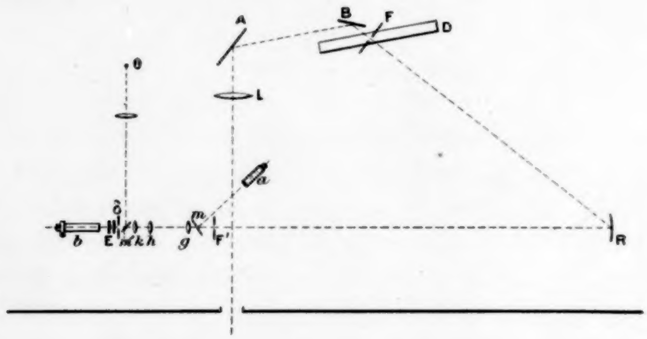

FIG. 4.

The arrangement adopted is shown in Fig. 4. The slit F' is at the center of curvature of the grating R; both are fixed. In order to make the spectrum pass across the slit F' without changing its focus, F must be moved along the circle whose diameter is RF'. In the region of the spectrum which we have employed, from $\lambda 460$ to $\lambda 650$, the displacement of F amounts to 1.90 m, on a circle 6.50 m in diameter. This arc of a circle does not differ greatly from its chord; the slit F was therefore mounted on the carriage of an optical bench D, occupying the position of the chord; this carriage can be given a slow motion by means of a screw controlled from a distance. As the arc of the circle does not coincide with its chord, the slit F must be moved a little in the direction FR in order to keep the spectrum in focus. This motion is effected with a rack and pinion.

Finally, the slit F must continue to receive light from the

solar beam in the direction *FR*. Fig. 4 indicates its path; after being reflected by the mirrors of a polar heliostat and a porte-lumiere, the beam enters the laboratory. At *A* it is reflected in a direction paralled to *D*. The mirror *B*, mounted on the same carriage with *F*, sends the beam toward the grating. In order to pass from one region of the spectrum to another, the carriage on which *B* and *F* are mounted is moved; both of these continue to receive the beam; it is only necessary to turn *D* very slightly (with a tangent screw) in order to keep the beam on *R*; this rotation is negligible when the displacement is small.

A real image of the Sun must be projected upon the illuminating slit *F*; for this purpose a lens *L* of 3.50 m focal length, which gives an image about 3 cm in diameter, is interposed in the path of the light.

If it is desired to displace the spectrum by a small amount (a few $\mu\mu$) it is only necessary to move the carriage. If the required displacement is great it is also necessary to move *L*, rotate *B*, and readjust the focus by sliding *F* in the direction *FR*. Only the last of these adjustment must be made with precision. There is no difficulty in accomplishing it by moving *F'* slightly in the direction *F'R* with a rack and pinion. In any case, the beam which emerges from *F'* remains fixed in position.

The slit *F'* can be removed from its support and the spectrum examined by means of the telescope *a* and the mirror *m*, which can also be removed. The observer at the telescope *a* has within reach the rod with which the carriage is moved. In order to bring upon *F'* the line which is to be measured we proceed as follows: *F* is made very narrow (about 0.05 mm) so as to give a perfectly sharp spectrum. The telescope *a* is focused on *F'*, and the cross-hairs set on the slit. This slit is then removed, permitting the real image of the spectrum reflected in the mirror to be seen. Without changing the adjustments of the telescope the focusing is accomplished by moving *F*. It is then possible, with the aid of Rowland's map, to recognize the lines and to select the one which is to be measured. The slit *F* is then widened (to about 0.5 mm). The lines remain visible,

though ill defined. By moving the carriage the one selected is set on the cross-hairs of the telescope; finally F' is replaced and the mirror m removed.

The beam which emerges from F' is to be utilized in the interference apparatus. This beam is comparatively faint and of small angular width, and must therefore be utilized in the best possible manner. This beam is limited by the area of the grating and the narrow opening of the slit. Furthermore, after passing through F', the beam must (1) pass through the small diaphragm which covers the standard; (2) illuminate, in a telescope focused for parallel rays, a surface large enough to show the first two or three rings; a field of from 2° to 3° is sufficient; a larger field would be unsuitable, in view of the faintness of the light.

To realize these conditions, the natural course was to place between F' and the standard an optical system which projects upon the diaphragm an image of the slit as small as possible and which gives in the telescope focused for parallel rays an image of the grating. It is impossible to satisfy both these conditions with a single lens; with two it can be done in an infinite number of ways. The optical system thus constituted must have one of its focal planes coincident with the grating. If Φ is the focal length of the system, a and b the height and width of the grating, the field will be a rectangle of angular extent $\frac{a}{\Phi}$, $\frac{b}{\Phi}$. If r is the radius of curvature of the grating (distance RF'), the image of the slit will be defined thus: $\frac{\text{image}}{\text{object}}=\frac{\Phi}{r}$. Thus, though the problem proposed is soluble in an infinite number of ways, all the solutions are equivalent. Only the focal distance of the system is involved. Φ would be selected as 3.20 m or 1.10 m, depending upon whether the settings on the rings are to be made in the horizontal or the vertical direction. In the first case the image of the slit would be too large, in the second, the angular aperture of the beam in the horizontal direction would be too large and light would be lost.

A much better result may be obtained with a system whose

focal lengths are unequal in two directions, so as to give in the vertical direction a large field and a small image of the slit (Φ small), and an enlarged image of the slit, with a small field, in the horizontal direction (Φ large). This may be accomplished by a combination of cylindrical lenses.

Let us first take two cylindrical lenses, with generatrix horizontal, forming a system which has one focal plane at the grating, a focal length Φ, and giving for a point at F' a horizontal focal line on the diaphragm δ. This system gives horizontal focal lines and consequently limits the images in the vertical direction. Now add two other cylindrical lenses, generatrix vertical, which satisfy the same conditions, but have a focal length Φ'. These limit the images in the horizontal direction without changing the foci of the preceding system. The field will thus be a rectangle whose angular height and width will be $\frac{a}{\Phi}$ and $\frac{b}{\Phi}$; both may thus be chosen by suitably determining Φ and Φ'.

For the angular apertures $150'$ was chosen in the vertical direction and $60'$ in the horizontal direction, which gives

$$\Phi = 1.10\,\text{m} \ , \quad \Phi' = 8\,\text{m} \ .$$

The dimensions of the image of the slit will be defined thus:

In the vertical direction, $\dfrac{\text{image}}{\text{object}} = \dfrac{1.1}{6.5} = 0.17$.

In the horizontal direction, $\dfrac{\text{image}}{\text{object}} = \dfrac{8}{6.5} = 1.2$.

This optical system, composed of four cylindrical lenses, has the following properties: Every point on the grating has an image at infinity (since the two focal lines corresponding to a point are at infinity), but the image of a circle traced on the grating is an ellipse greatly elongated in the vertical direction (ratio of the axes, 7.3); the image in the telescope of the horizontally elongated rectangle of the grating is a rectangle elongated vertically. Every point on the slit has also an image on the diaphragm, and the image of a circle traced on the slit is an ellipse elongated horizontally (ratio of the axes, 7.3); the

image of the slit thus tends toward a square form. In this way the light is perfectly utilized.

These four lenses, which might be defined in an infinite number of ways, were chosen as follows: Two of the cylindrical lenses, occupying the same point, are replaced by a spherical lens and a cylindrical lens. The two others are then determined.

At *g* there is placed a spherical lens of + 1 diopter and a cylindrical lens of + 2 diopters, generatrix horizontal.

At *h* a cylindrical lens of + 2.5 diopters, generatrix vertical.

At *k* a cylindrical lens of + 7 diopters, generatrix horizontal.

At *E* is the standard, at δ the diaphragm, at *b* the telescope used to measure the diameter of the fringes.

The light from the cadmium tube θ may also be thrown on the diaphragm by means of the mirror m', which can be removed; an image of this tube is projected upon δ.

Selection of the solar lines.—In selecting solar lines for measurement, a few of those which give the most perfect fringes were chosen. The strongest lines are too broad, their wave-length is uncertain, and the fringes diffuse. Certain very faint lines give very fine fringes, but these are hardly visible. The lines which seem to us to give the best results are, for the most part, those whose intensities lie between 5 and 7 in Rowland's table.[1] So far as possible isolated lines have been selected in order to avoid confusion.

For the interference apparatus we have employed exclusively the 2.5 mm standard. Many lines give fringes which are clearly visible with greater thicknesses, but even with the 5 mm standard these fringes are diffuse and what is gained by the higher order is lost in the precision of the settings.

Order in a measurement.—One of the observers, stationed at telescope *a*, sets the lines on the slit F'. Without changing his position, he controls the motion of the carriage required to displace the spectrum, moves the slit F' in the direction $F'R$ for

[1] "Preliminary Table of Solar Spectrum Wave-Lengths," ASTROPHYSICAL JOURNAL, 1–5, *passim*. The numbers which denote the intensity of the lines increase with the intensity.

the purpose of focusing, and removes and replaces the slit F' as well as the mirror m. This last change is made when the line is on the slit; the light can then pass to the interference apparatus. The second observer, stationed at telescope b, makes the diameter measurements. He manipulates the mirror m' and the interrupter of the commutator for illuminating the cadmium tube. The diameter measurements are made in the order indicated above, and the calculations are effected in the same manner.

It is highly important to avoid errors due to variations of wave-length arising from the solar rotation. In passing from the center to the limb at the solar equator, the wave-length varies $\frac{1}{150,000}$ of its value; in this way an error seven times as great as the accidental error might be committed. The rings show an easily visible contraction or expansion in passing from one limb to the other at the solar equator. These variations can be avoided by projecting upon the slit an image of the Sun's polar diameter. In the conditions of our experiment this diameter was very nearly vertical; it was only necessary to center the Sun's image on the middle of the slit.

Results.—We have measured the wave-lengths of 33 lines of the solar spectrum distributed between 460 $\mu\mu$ and 650 $\mu\mu$. Table III gives for each line the wave-length λ_R given by Rowland; the wave-length λ found by us; the ratio $\frac{\lambda_R}{\lambda}$ of the first number to the second.

Fig. 5 is plotted from the results in the table; the abscissae are wave-lengths and the ordinates are values of the ratio $\frac{\lambda_R}{\lambda}$.

1. One remark is unavoidable at the outset: the points are not scattered by chance, but are distributed regularly on a curve. The deviation between the points and the ordinates of the curve does not exceed a millionth. This shows that the relative wave-lengths of lines in the same region have been determined by Rowland with an exactness of this order, a precision which we have obtained, if not surpassed, in the comparison of any given line with the green line of cadmium.

2. If Rowland's scale were normal these wave-lengths would

differ from ours only by a constant factor; the ratio $\frac{\lambda_R}{\lambda}$ would be constant, and the above curve would have been a horizontal straight line. It is evident that this is not the case, the systematic variations of the ratio being enormously greater than the accidental deviations. Between wave-lengths $525\mu\mu$ and $555\mu\mu$ the ratio varies about 8×10^{-6}. On each side of the mean value the deviations amount to 4×10^{-6} or, in terms of the wave-length, $0.002\mu\mu$.

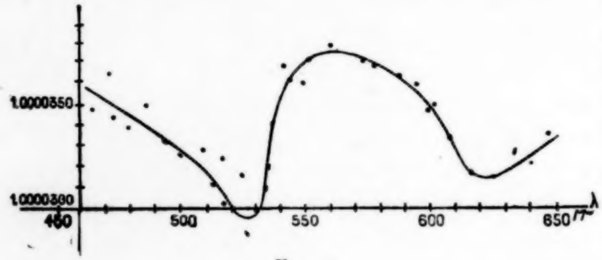

FIG. 5.

The above results confirm what has been said on the properties of gratings: as an apparatus for interpolation the grating gives the millionth, but for widely separated lines the relative wave-lengths measured by Rowland may show deviations eight times as great.

All modern spectroscopic measurements which are based upon Rowland's values contain the same errors. In order to correct them and at the same time to reduce them to Michelson and Benoit's standard, it is only necessary to divide the numbers by the ratio $\frac{\lambda_R}{\lambda}$ read from the curve (Fig. 5).

The following tables give:

Table I, the wave-lengths of certain metallic lines previously determined by us by the method of coincidences.[1]

Table II, the wave-lengths of iron lines in the arc at atmospheric pressure.

Table III, the wave-lengths of solar lines.

[1] *Journal de Physique* (3), **9**, 369, 1900 and *Comptes Rendus*, **130**, 406 and 492, 1900.

TABLE I.

VARIOUS SOURCES.

Metal	Source	Wave-length
Mercury	arc in vacuo	435.8343
Zinc	vibrator in vacuo	468.0138
	"	472.2164
	"	481.0535
Copper	"	510.5543
	"	515.3251
Silver	"	520.9081
Copper	"	521.8202
Mercury	tube	546.07424
Silver	vibrator in vacuo	546.5489
Mercury	tube	576.95984
Copper	vibrator in vacuo	578.2090
	"	578.2159
Mercury	tube	579.06593
Sodium	flame	588.9965
	"	589.5932
Zinc	vibrator in vacuo	636.2345
Lithium	flame	670.7846

TABLE II.

IRON LINES.

473.6785	523.2954	558.6775	623.0733
485.9763	530.2321	561.5657	649.4992
500.1887	543.4525	576.3023	
508.3345	550.6783	606.5489	

TABLE III.

SOLAR LINES.

λ_R	λ	$\frac{\lambda_R}{\lambda}$	λ_R	λ	$\frac{\lambda_R}{\lambda}$
464.3645	464.3483	1.0000349	549.7735	549.7536	1.0000362
470.5131	470.4960	363	550.7000	550.6794	374
473.6963	473.6800	344	558.6991	558.6778	381
478.3613	478.3449	340	571.5308	571.5095	373
485.9928	485.9758	350	476.3218	576.3004	371
492.4107	492.3943	333	586.2582	586.2368	365
500.2044	500.1881	326	593.4881	593.4666	362
509.0954	509.0787	328	598.7290	598.7081	349
512.3899	512.3739	312	601.6861	601.6650	351
517.1778	517.1622	302	606.5709	606.5506	335
524.7229	524.7063	316	615.1834	615.1639	317
524.7737	524.7587	286	623.0943	523.0746	316
534.0121	533.9956	309	632.2907	632.2700	327
534.5991	534.5820	320	633.5554	633.5346	328
536.7669	536.7485	343	640.8233	640.8027	321
541.0000	540.9800	370	647.1885	647.1666	338
543.4740	543.4544	361			

THE INFLUENCE OF ATMOSPHERES OF NITROGEN AND HYDROGEN ON THE ARC SPECTRA OF IRON, ZINC, MAGNESIUM, AND TIN COMPARED WITH THE INFLUENCE OF AN ATMOSPHERE OF AMMONIA.

By ROYAL A. PORTER.

IT has been frequently assumed that chemical reactions in the electric arc have a considerable influence on the character of its radiations.[1] It seems not unreasonable to expect that the oxidation of the electrodes at high temperature in air would tend to increase the intensity over that obtained when the arc is operated in nitrogen alone. This presumed higher temperature might be sufficient to produce atomic vibrations entirely distinct from the vibrations at a lower temperature. If the atmosphere does have any such influence, the effect might be apparent in the spectrum of the arc.

In the case of a hydrogen atmosphere[2] the most marked effects on the arc spectrum of iron, zinc, magnesium, and tin have been found to be a general diminution of intensity and a change of relative intensity among the lines. The lines relatively enhanced by hydrogen are spark lines.

Liveing and Dewar[3] have noted the effect of atmospheres of hydrogen and nitrogen on a number of lines in the magnesium arc and spark and of these and other atmospheres on the cyanogen[4] bands in the carbon arc, but so far as I am aware no extensive study has been made of the influence of nitrogen and ammonia on the arc spectrum of metals. From a study of the

[1] LIVEING and DEWAR, *Proc. Roy. Soc.*, **30**, 161, 1880; **32**, 192, 1881.

O. H. BASQUIN, ASTROPHYSICAL JOURNAL, **14**, 11, 12, 1901.

A. S. KING, ASTROPHYSICAL JOURNAL, **14**, 329, 330, 1901.

[2] H. CREW, ASTROPHYSICAL JOURNAL, **12**, 167, 1900; LIVEING and DEWAR, *Proc. Roy. Soc.*, **32**, 192, 402, 403, 1881.

[3] *Proc. Roy. Soc.*, **32**, 189–203, 1881.

[4] *Ibid.*, **30**, 152–162.

effects of hydrogen and nitrogen on certain spark lines of magnesium, Liveing and Dewar[1] were led to remark that "it is possible that the atmosphere may, besides the resistance it offers to the discharge, in some degree affect the vibrations of the metallic particles." This conclusion in regard to the spark seems to have been borne out with reference to the arc by the results obtained with a hydrogen atmosphere.[2]

Nitrogen resembling hydrogen in its inability to combine directly with metals, it seemed reasonable to expect that it would have a very similar effect on the iron, zinc, and tin arcs. As magnesium[3] combines directly with nitrogen, a different effect might be expected in the magnesium arc.

METHOD AND APPARATUS.[4]

In order to eliminate the effect of gases other than the one whose effect was being studied, the enclosed rotating metallic arc[5] with "chemically pure" zinc, magnesium, and tin electrodes was used. The iron used, however, was "commercial." The speed of the rotating electrode was approximately 1100 revolutions per minute. Excepting in two instances noted later, the arc was operated by a 104-volt alternating current.

The spectrum was photographed with a Rowland ten-foot concave grating, four exposures being made on each plate as shown in the diagram.

Long Exposure, Arc in Air
Long Exposure, Arc in Gas
Short Exposure, Arc in Gas
Short Exposure, Arc in Air

[1] *Proc. Roy Soc.*, **32**, 203, 1881. [2] ASTROPHYSICAL JOURNAL, **12**, 167–175, 1900.

[3] LIVEING and DEWAR, *Proc. Roy. Soc.*, **32**, 161, 1881.

[4] The funds to meet the expense of this experiment were kindly appropriated by the committee in charge of the Rumford fund of the American Academy of Arts and Sciences.

[5] CREW and TATNALL, *Phil. Mag.*, **38**, 379–386, 1894.

Near the top of the plate was photographed the spectrum of the arc in air; just below this was photographed the same arc operated in an atmosphere of the gas being studied; below this, again, was made a short exposure with the arc in the same gas; and at the bottom, a short exposure with the arc in air. The exposures were so timed as to make the intensities of the two inner photographs intermediate between the intensities of the two outer ones. By comparison one can readily determine from such a plate whether any change in intensity is due to length of exposure or to change of atmosphere. This plan of making four exposures on each plate has some additional advantages: (1) it affords a test of uniformity of results; (2) the two short exposures allow an easier comparison of lines so strong as to be ordinarily overexposed; (3) the second exposure in the gas being made immediately after the first, and without opening the "hood" of the arc, a photograph is obtained of the spectrum of the arc in its final atmosphere, that is, in an atmosphere which includes the gaseous products of the arc, if there be any.

The effect of hydrogen was taken from the results published by Professor Crew and from the original plates obtained by him. These negatives were made under the same conditions as here described, except in a different atmosphere and with a direct current. The change from a direct to an alternating current, however, produces no effect on the spectrum. In the case of the tin arc, less dust is produced when a direct current is used, and I have therefore employed such a current in photographing the spectrum of tin in ammonia.

The hydrogen atmosphere was obtained by the electrolysis of acidulated water. The ammonia used was taken from a drum of compressed ammonia gas, such as is used in refrigeration. Nitrogen was generated by the reaction of ammonium sulphate and sodium nitrite solutions. Professor J. H. Long kindly suggested an arrangement of the nitrogen generator by which air was excluded from the entire system throughout the work.

Traces of oxygen were removed by pyrogallic acid, while

the water vapor was taken out by passage through concentrated sulphuric acid and over phosphorus pentoxide.

Before commencing an exposure a stream of nitrogen was kept flowing through the hood for at least twenty minutes. The same plan was used in filling the hood with ammonia. During an exposure a stream of the gas was kept flowing through the hood, both for the purpose of keeping a fresh supply of the gas about the arc and to drive out the dust which formed.

The spectra of the following four metals have been examined photographically in the region lying between λ 2300 and λ 5300.

MAGNESIUM.

The ordinary arc lines of magnesium seem to be almost wholly unaffected by substituting nitrogen for air. The heavy spark line at λ 4481, which appears also in the spectrum of the rotating arc, is reduced by nitrogen to about one-fifth its intensity in air. Naturally, the magnesium oxide fluting at λ 5007 is practically blotted out in nitrogen and greatly intensified in oxygen. On the other hand, no new lines make their appearance in nitrogen.

The intensity and reversal of the characteristic line at λ 2852, which are so strongly affected by hydrogen and ammonia, are unaffected by nitrogen, while the sharp line at λ 4571 is slightly reduced by hydrogen and ammonia, although not changed by nitrogen.

The magnesium-hydrogen fluting beginning at λ 5210 discovered by Liveing and Dewar[1] appears in ammonia, as does also the F line of hydrogen. Fairly intense hazy lines at approximately λ 4580, 4434, 4430, and 4390 also appear in ammonia. These lines are apparently intensified by oxygen also. By nitrogen they are unaffected. A very faint trace of these lines can be seen in air. I have not yet succeeded in identifying them.

TIN.

Of the four metals studied, tin is the one whose spectrum is most modified by changes of atmosphere. In air, nitrogen, and

[1] *Proc. Roy. Soc.*, **27**, 494–496; **30**, 93–99.

oxygen the tin arc works well. But in hydrogen and ammonia the arc is very short and is maintained with difficulty.

The intensity of the tin arc in nitrogen is estimated at one-third of its intensity in air; while in ammonia its intensity is not more than one-twentieth of its intensity in air, and in pure hydrogen it is even less.

Not only is the average intensity of the tin spectrum strongly affected, but also the character and relative intensity of its individual lines. In Kayser and Runge's table of wave-lengths of the tin arc more than forty lines are described as reversed. Many of these reversals show very clearly on my plates. When the arc is surrounded with nitrogen, however, some of these lines appear to be doubly reversed, the rest not reversed at all. On the other hand many of the lines that are reversed in air appear to have their reversal widened by ammonia. These reversals are not affected by an atmosphere of oxygen.

The two very strong spark lines at $\lambda 3351$ and $\lambda 3282$, appearing also in the spectrum of the rotating arc, have their relative intensities decreased by nitrogen. But by ammonia they are relatively enhanced at least twenty times. In the arc in air, these two lines are barely visible; but in ammonia they become two of the most prominent lines of the spectrum. These two lines are similarly affected by hydrogen. The spark line at $\lambda 2368$ is also intensified in ammonia, but less than the two preceding. The spark lines of wave-lengths 2449, 3471, 3539, 3574, appear plainly in ammonia and hydrogen but not at all in air or nitrogen. They are more enhanced by hydrogen than by ammonia. At $\lambda 3360$ and $\lambda 3370$ appear two unidentified lines of intensity 6 and 3 (on a scale increasing from 1 to 10), respectively, in ammonia, and not quite so intense in nitrogen, which can scarcely be detected in air and hydrogen. These same lines appear also in the magnesium and zinc arcs in air, nitrogen, and ammonia; but oxygen has the effect of immensely weakening both of them in the tin and magnesium arcs. $\lambda 3370$ appears in the tin spark, but $\lambda 3360$ does not.

ZINC.

The average intensity of the lines of the zinc arc spectrum is reduced approximately one-half by nitrogen. The width of reversed lines is not affected.

The only zinc lines that suffer a disproportionate reduction by an atmosphere of nitrogen are $\lambda 2558$ and $\lambda 2502$. These, with $\lambda 5182$, are the lines which are enhanced by hydrogen. They are strong spark lines. In ammonia their intensity is two or three times as great as it is in air, although the average intensity of the zinc spectrum is diminished perhaps five times by ammonia.

IRON.

The substitution of nitrogen for air about the iron arc produces very little change in the spectrum. The general intensity is not altered. Compared with the number of lines relatively affected by hydrogen the number of iron lines affected by nitrogen is small, as has been found to be the case with other metals. Many of the lines that are affected by nitrogen are impurity lines. Of fifty iron lines between $\lambda 3660$ and $\lambda 4060$ that are markedly enhanced by hydrogen, six are distinctly reduced by nitrogen.

On examination of the region between $\lambda 3600$ and $\lambda 4600$ I found twenty-six lines that are particularly affected by nitrogen. A few others are slightly changed. Of these twenty-six lines seventeen are from two to ten times as strong in nitrogen as in air. But these seventeen lines were all found to be due to impurities, fifteen belonging to manganese, one to chromium, and one to cobalt. The remaining nine lines are *reduced* by nitrogen to from one-half to one-tenth their intensity in air. Six of these nine lines belong to the spectrum of the iron spark.

EFFECT OF EXCLUDING NITROGEN.

The effect of pure nitrogen being so slight, it seemed possible that the presence of so large a percentage of nitrogen in the air about the ordinary arc might account for the smallness of the change. I therefore attempted to determine the effect of nitro-

gen by a process of exclusion. This was done by substituting for air an atmosphere of commercial oxygen taken from an ordinary stereopticon gas-cylinder. A stream of this oxygen was kept flowing through the hood of the arc.

If the chemical affinity of the electrodes for the atmosphere has any effect on the spectrum, one might certainly expect this effect to be exhibited when such easily oxidizable metals as iron, magnesium, and tin are employed in the atmosphere of oxygen.

Mr. A. S. King[1] found that the metallic lines were intensified by increasing the supply of oxygen about the *carbon* arc. But just the contrary seems to be true with reference to the *metallic* arc. A current as large as ten amperes was tried with chemically pure magnesium electrodes, but the oxygen had little effect on the working of the arc or the appearance of the so-called "flame." To the eye the "flame," especially with iron electrodes, appears to be less blue and more yellow.

Of eighteen exposures made with the iron, tin, and magnesium arcs in oxygen, all but one show a greater average intensity for the same length of exposure in air than in oxygen. This is not the only respect in which the action of oxygen on the metallic arc resembles that of hydrogen and ammonia. The metallic lines that have been noted as being relatively enhanced or reduced by hydrogen are precisely the ones which are so affected by oxygen. The changes produced by oxygen are not so great as those produced by hydrogen, but they are in the same direction.

SUMMARY.

The results of these experiments may therefore be summarized as follows:

1. The average intensity of the iron and magnesium arcs is not changed by substituting pure nitrogen for air as an atmosphere. The average intensity of the zinc and tin arcs is reduced two or three times by nitrogen. By hydrogen the average intensities are reduced from five to twenty times as much as by nitrogen. Ammonia apparently does not produce quite so great a reduction as does hydrogen.

[1] ASTROPHYSICAL JOURNAL, 14, 329, 1901.

2. The relative intensities of many lines depend upon the atmosphere. The lines that are relatively reduced by nitrogen are spark lines. As a rule these lines are relatively enhanced by hydrogen or ammonia.

3. The influence of ammonia on intensities and reversals is intermediate between that of nitrogen and hydrogen, and in general it seems true that *the effect of ammonia is approximately equal to the sum of the effects of its constituents*. This, in fact, is the particular point which I had in mind to determine when I began these experiments.

4. *The influence of oxygen is similar to that of hydrogen.*

5. Nitrogen affects the reversed lines of tin by either destroying the reversal or producing faint double reversals.

These results seem sufficient to show that *the readiness of an atmosphere to form chemical union with the electrodes under ordinary conditions is a very small, probably insignificant, factor in determining the intensity of the arc. The intensity appears to be due to electrical causes rather than to chemical reactions.*

Some experiments have been performed by Professor Basquin[1] which seem to confirm the theory that the intensification of spark lines in hydrogen is caused by the increased resistance due to the hydrogen atmosphere about the arc. This greater resistance has been attributed to the absence of chemical reaction in the hydrogen arc. Such a change of resistance in the products of the arc may explain the phenomena which occur in hydrogen. But if the resistance of the arc depends on the reactions in it this fact makes it difficult to see how the spark lines can be intensified by an atmosphere of oxygen.

This work was done under the direction of Professor Crew, who, in fact, himself began it, and to whom I am indebted for continued advice and assistance.

NORTHWESTERN UNIVERSITY,
Evanston, Ill.,
April 22, 1902.

[1] ASTROPHYSICAL JOURNAL, **14**, 14–17, 1901.

ON A NUMERICAL RELATION BETWEEN LIGHT AND GRAVITATION.

By Victor Wellmann.

Since Helmholtz, in his epoch-making treatise on "Die Erhaltung der Kraft," demonstrated the unity and the mutual capacity for transformation of all nature-forces, and pointed out the task of natural science in the words, "The phenomena of nature must be referred back to motions of matter with unchangeable energy of motion, depending on spacial relations alone," attempts have been made in the spheres of physics and chemistry to explain the phenomena through the collision of masses in motion. In astronomy, also, various attempts have been made to explain the universal attractive power of masses, acting according to Newton's law, through the gas-pressure of the supposedly gaseous interstellar medium, in order to escape from the hypothesis of an "*actio in distans*," in itself inexplicable and impossible.

As a matter of fact, universal gravitation allows of a simple, unconstrained explanation by means of the pressure of the interstellar medium under the application of the laws of the kinetic theory of gases. Yet, at the same time, we see that the law formulated by Newton, $K = \frac{k}{r^2}$, is only strictly accurate under the limiting condition that the two reciprocally attracting bodies do not change their reciprocal position; in this case the law of Newton must be modified to suit the condition that the force depends not only on the reciprocal distance, but also on the motions of the bodies in relation to one another. This of course also alters the laws of motion which are valid for the heavenly bodies, so that an examination of them is of great interest. Thus Professor Laves[1] has subjected the motional

[1] "The Ten Integrals of the Problem of *n*-Bodies," *Astronomical Journal*, **19**, 97, 1898.

equations of the *n*-bodies problem to a very interesting examination, in which he applies especially the electro-dynamical laws of Weber, Riemann, and Clausius.

In the following lines I intend—on the basis of an analogous law of attraction, according to which the causes of gravitation are likewise referred back to the pressure of the interstellar medium—to point out a numerical relation between light and gravitation, which ought to show the way to an explanation of the connection of these two forces.

Let δ_0 be the symbol of the "relative density" of the interstellar medium, that is to say, of the mass of ether, which in the unit of time passes the unit of surface, and let V be the velocity of the single particles of this mass; then Newton's law of gravitation will be thus expressed:[1]

$$K = \frac{\delta_0 V^2}{r^2}, \tag{1}$$

from which is derived Gauss's constant, $k = \delta_0 V^2$.

Suppose that the attracted body is not at rest, but moves in relation to the attracting body with the radial component of velocity $\frac{dr}{dt}$, then the number of the interstellar particles which meet the attracted body in the unit of time will be:

$$\delta = \delta_0 \left(\frac{V - \frac{dr}{dt}}{V} \right).$$

At the same time in (1) we must put $\left(V - \frac{dr}{dt}\right)$ for V. Thus we have for the attractive force of masses in motion the equation:

$$K = \frac{\delta_0 V^2}{r^2} \left(1 - \frac{1}{V} \frac{dr}{dt} \right)^3;$$

or, as we can, on account of the smallness of the factor $\frac{1}{V}$, sub-

[1] V. WELLMANN, "Ueber die Ursachen der Gravitation," *A. N.*, **144**, 121, 1897; "Ueber das Newton'sche Gravitations-Gesetz," *A. N.*, **148**, 169, 1899; "Ueber den Einfluss des widerstehenden Mittels auf die Planetenbahnen," *A. N.*, **148**, 297, 1899.

stitute for $\frac{dr}{dt}$ the value which results from the Newtonian law of attraction, $\frac{dr}{dt}=\frac{k}{r}$, we have the result:

$$K=\frac{k}{r^2}\left(1-\frac{k}{rV}\right)^3. \qquad (2)$$

That this divergence from the law of Newton has not been demonstrable by observation is not surprising, considering the smallness of the factor $\frac{k}{V}$.

Suppose we take the imaginary case $\frac{dr}{dt}=1$ (of course with all magnitudes reckoned in Gauss's units), *i. e.*, suppose we consider an imaginary planet at the distance $r=k$, and take for the velocity of light in vacuo $V=173{,}492$,[1] we have the "factor of gravitation:"

$$F=\left(1-\frac{k}{rV}\right)^3_{r=k}=0.98280755\ .$$

Hence the complementary magnitude:

$$K'=1-\left(1-\frac{k}{rV}\right)^3_{r=k}=0.01719245\ ,$$

while the Gaussian constant is:

$$k=0.01720210\ .$$

To the difference $k-K'=\Delta k=0.00000965$, if it were actually to exist, would correspond an inaccuracy of 0."00497 in the solar parallax, or an inaccuracy of 169 kilometers in the velocity of light, while the corresponding errors given by Harkness amount to 0."00567 and 80 kilometers.

We see that our "factor of gravitation," derived from the velocity of light, agrees with the constant of Gauss within the required limits of accuracy, whence the theorem:

"At the distance k the 'factor of gravitation' is equal to the complement of Gauss's constant k."

It might be objected to this calculation, that the dimensions of the equations for K' and k are different, and hence that these two equations cannot be compared with each other.

[1] WILLIAM HARKNESS, "The Solar Parallax and its Related Constants," *Washington Observations* for 1885.

This objection at first seems certainly to be to some extent justified; but it is, however, purely of a formal nature. Equations of different dimensions can very well stand in causal connection with one another, in spite of this difference. Thus, for example, the linear velocity of the Earth, $w = \frac{2\pi r}{T} = 0.01720213$, stands unquestionably in causal connection with the Gaussian constant k and agrees with it within the decimals above stated, although its dimensions are entirely different from those of the Gaussian constant, $k = \frac{2\pi a^{\frac{3}{2}}}{T\sqrt{M+m}}$. The demand that our constant K' shall agree with the dimensions of k' is in general absolutely unwarranted. The dimensions of k are derived from Kepler's law, which is itself based on the strict validity of Newton's law, whereas our considerations rest on the basis of another law.

If we grant that gravitation depends not only on the mass and the reciprocal distance of the bodies, but also on their relative velocity, we must of course also assign to it other dimensions, as in this case Kepler's law is no longer strictly valid. Besides this, it is more correct to consider the constant of Gauss as a so-called "factor of proportionality,"[1] *i. e.*, as without dimensions, just as our factor of gravitation is without dimensions.

Another more lucid formula, which at the same time shows that our relation can be expressed as universally valid, independently of the limiting condition $r = k$, can be given to our relation by introducing, instead of the radial component of velocity, the linear velocity $\frac{ds}{dt}$.

Hence from $\frac{ds}{dt} = \frac{k}{\sqrt{r}}$ follows:

$$\left(1 - \frac{ds}{dt}\frac{\sqrt{r}}{kV}\right)^3 = 1 - k\,, \tag{3}$$

or

$$V = \frac{1}{1 - \sqrt[3]{1-k}}\,. \tag{4}$$

[1] WEINSTEIN, *Handbuch der physikalischen Maassbestimmungen*, Bd. II.

If we calculate by this formula the velocity of light from the Gaussian constant of attraction, the result is $V = 173,394$ in Gauss's units, or a value in so close agreement with that given by Harkness that a merely accidental coincidence of the two values is out of the question, and a causal connection between light and gravitation seems to be expressed in these figures.

SPECTROGRAPHIC MEASURES OF THE VELOCITIES OF GASEOUS NEBULÆ.[1]

By J. Hartmann.

Although the bright line spectra of the nebulæ have repeatedly been photographed, so far as I know no attempt has yet been made to secure radial velocity determinations from the plates obtained. The cause for this may be that the spectrograms did not have a comparison spectrum, or that the small scale of the photographs precluded accurate measurement. The epoch-making work of Keeler[2] did, indeed, furnish relatively accurate velocities for fourteen of the brighter nebulæ; but anyone acquainted with the extreme difficulty of making such visual measures, must admit that in this case, as in others, a very considerable increase of accuracy is to be obtained by the application of modern spectrographic methods. Such a result I regard as of extreme importance. For, if it becomes possible to secure velocity determinations for the nebulæ with an error not exceeding a few tenths of a kilometer, we may certainly expect to find relative motion within every one of these objects; and the detailed study of such motions will prove of fundamental importance not only for our knowledge of these systems, but for our ideas of cosmogony in general.

A somewhat casual photograph of the planetary nebula *G.C.* 4390, which I made with the 80 cm photographic refractor, gave a very strong image of this object after an exposure of but fifteen minutes, a result which led me to the conclusion that it must be possible to obtain photographs of the spectra of the brightest of the nebulæ with the spectrographs at hand.

In my investigations I have made use of Spectrographs I and

[1] Translated from the *Sitzungsberichte der k. Acad. der Wiss. zu Berlin.* Session of February 27, 1902.

[2] "Spectroscopic Observations of Nebulæ." *Publications of the Lick Observatory*, 3, 1894.

III, which were constructed for the 80 cm refractor. Spectrograph I has a single flint glass prism of 60° angle, and a collimator of 530 mm and camera of 720 mm focal length. This instrument, on account of its low dispersion and long camera, cannot be regarded as exactly suitable for the spectra of nebulæ. It has, however, the advantage of giving a sharp image of the entire extent of spectrum from λ3600 to λ5900, and so has enabled me to photograph $H\gamma$ at the same time with the green nebular line. Spectrograph III, in the form in which I have used it here, is better adapted for the spectra of nebulæ. It has a collimator of 480 mm focal length, and three flint glass prisms of 63° each. In place of the camera of 560 mm focal length, in ordinary use for stellar spectra, a shorter one of 410 mm may be attached, giving a corresponding increase of intensity at the focus. This camera, whose objective gives but a short extent of the spectrum in sharp focus, I adjusted in such a way that the excellent group of iron lines extending from λ4860 to λ5006 fell in good focus at the center of the plate, and so furnished a very convenient comparison spectrum for the three brightest nebular lines. To avoid disturbing the excellent adjustment of the instrument, the prisms were left at their setting for minimum deviation on $H\gamma$.

With these two instruments, and the assistance of Dr. Ludendorff, I secured the following plates:

TABLE I.

Instrument and No. of Plate	1901	Central European Time	Nebula	Length of exposure
I 120…….	September 23	8^h 35^m	*G.C.* 4390	100^m
I 123…….	" 24	8 25	*G.C.* 4390	90
I 127…….	" 25	9 4	*G.C.* 4373	120
I 144…….	October 31	8 45	*N.G.C.* 7027	120
III 389…….	September 30	10 30	*G.C.* 4373	270
III 390…….	October 1	8 20	*G.C.* 4390	180
III 392…….	" 3	8 25	*G.C.* 4390	210

The following should be noted of the individual plates:

I 120. The slit was set at position angle 90°, and the edge

of the nebula kept on the middle of the slit. The lines consequently do not occupy the full length of the slit, but end at its center. They are very weak, and only the chief nebular line (λ 5007), which I will denote by N_1, admits of at all accurate measurement. Traces of the second nebular line N_2 (λ 4959), and of $H\beta$ are present.

I 123. A very good plate taken at the center of the nebula. N_1 and N_2 strong; $H\beta$ also well measurable. $H\gamma$ very weak and difficult to measure.

I 127. A rather weak plate. N_1 alone well measurable; N_2, $H\beta$, and $H\gamma$ very difficult and uncertain.

I 144. Only N_1 and N_2 measurable. Faint traces of $H\beta$ and $H\gamma$.

III 389. A very weak negative. N_1 alone measurable, and that with difficulty.

III 390. N_1 strong, N_2 also well measurable. A very weak trace of $H\beta$, which appears so faint under the microscope as to be measurable only with great difficulty.

III 392. N_1 extremely strong, N_2 strong, $H\beta$ rather weak, but still well measurable.

The arc spectrum of iron was used as the comparison spectrum, and a glass plate was interposed between the arc and slit to diffuse the light. I adopted the following wave-lengths from Rowland's table of the solar spectrum:

4294.30	4376.11	4903.50
4315.26	4736.96	4920.68
4337.22	4859.93	4957.67 [1]
4352.91	4878.41	5006.12

I have measured each plate four times, namely, in both directions (violet right, and violet left) from two positions of the micrometer screw differing from each other by 0.5 revolution. The progressive screw errors have been accurately determined, and are allowed for in the reduction. A double thread was used for the settings upon the lines, and the measures could be made with extreme accuracy whenever the lines were sufficiently dense. As the emission lines of the nebulæ have exactly

[1] A double line (see remarks below).

the same appearance as the lines of the comparison spectrum, the measures are much less liable to systematic errors of setting than in the case of stellar spectra with absorption lines.

In order to secure as accurate a value as possible for the best plate III 392, and to give it suitable weight in the series, I have made two sets of measures upon it, or eight measures in all. The two sets of measures are indicated in the table below by III 392a, and III 392b. The following wave-lengths were found for the lines measured upon the different plates:

TABLE II.

Plate	N_1	N_2	$H\beta$	$H\gamma$
I 120	5007.36	....	....	....
I 123	5007.25	4959.34	4861.71	4340.86
I 127	5006.10	4958.26	4860.58	4339.65
I 144	5007.44	4959.58	....	....
III 389	5005.89	....	....	....
III 390	5007.30	4959.46	4861.79	....
III 392 a	5007.31	4959.42	4861.79	....
III 392 b.....	5007.27	4959.43	4861.76	....

I have used these wave-lengths in the following way: From the measures of the hydrogen lines $H\beta$ and $H\gamma$, made on plates I 123, III 390, III 392a, and III 392b, the velocity of the nebula *G. C.* 4390 was first determined (Table III). With this velocity the wave-lengths of the lines N_1 and N_2 were next derived (Table IV); and finally with the use of these wave-lengths the velocities in Table V were obtained from all the values of Table II.

The hydrogen lines give the following determinations of the radial velocity of *G. C.* 4390.

TABLE III.

Plate	Line	Wave-length in Nebula	$d\lambda$	V_1 (In ref. to Earth)	Reduction to Sun	V (In ref. to Sun)
I 123...	$H\beta$	4861.71	+0.18	+11.1 km	−25.8 km	−14.7 km (½)
	$H\gamma$	4340.86	+0.23	+15.9	−25.8	− 9.9 (½)
III 390 ..	$H\beta$	4861.79	+0.26	+16.0	−25.7	− 9.7
III 392a .	$H\beta$	4861.79	+0.26	+16.0	−25.6	− 9.6
III 392b .	$H\beta$	4861.76	+0.23	+14.2	−25.6	−11.4

The weight of ½ was assigned to the values given by plate

I 123, on account of the lower degree of accuracy permitted by the weak dispersion of the instrument. As we see, the values of the velocity agree very well. The final result $V = 10.75$ km has a probable error of ± 0.56 km, while the final result for the *Orion* nebula, which Keeler found from 13 nights' visual measures of the $H\beta$ line, has a probable error of ± 1.29 km. The conclusion may well be drawn that the accuracy of Keeler's values has been surpassed, in spite of the rather unsuitable character of the apparatus which I have made use of in these preliminary investigations. To fully appreciate this photographic result we should bear in mind that the dispersion used by Keeler in the third and fourth orders of his grating would correspond to that given by 14 and 24 prisms, respectively. It is evident, therefore, that the photographic method, when used with an instrument especially constructed for this purpose, would be capable of giving considerably more accurate results. For the motion of the nebula in reference to the observer we find:

	I 123	III 390	III 392
Motion of nebula referred to Sun..........	−10.75 km	−10.75 km	−10.75 km
Orbital motion of Earth	+25.66	+25.53	+25.43
Rotation of Earth......................	+ 0.16	+ 0.18	+ 0.20
V_1	+15.07	+14.96	+14.88

These three velocities give a correction to the values of the wave-lengths of the two nebular lines N_1 and N_2 of -0.25 tenth-meters. Applying this we find from the apparent wave-lengths given in Table II the following true wave-lengths, freed from the influence of motion.

TABLE IV.

Plate	N_1	N_2
I 123	5007.00 (½)	4959.09 (½)
III 390	5007.05	4959.21
III 392 a	5007.06	4959.17
III 392 b	5007.02	4959.18
Mean........	5007.04	4959.17

The agreement of the independent values is, in this case as before, so excellent that their mean deserves confidence in spite of the small amount of observational material.

The value found by me for the wave-length of the chief nebular line N_1 agrees almost exactly with that of $\lambda = 5007.05 \pm 0.03$ derived by Keeler from his observations of the *Orion* nebula. But I find a considerably larger value for the wave-lengths of the second line, for which Keeler gives $\lambda = 4959.02 \pm 0.04$. The latter determination is based upon five comparisons of the nebular line with the double line $\lambda 4957.480$ and $\lambda 4957.785$ of the iron spectrum. Keeler, in his observations, did not see this line double, and so used as the wave-length of his comparison line the arithmetical mean 4957.63. In Rowland's solar spectrum table the two lines have the intensities 5 and 8. If we form the weighted mean with these intensities we find as the wave-length of the blend formed by the two lines 4957.67, which is the value I adopted in my reductions. Using this result we find that Keeler's wave-length for the second nebular line becomes 4939.06, a value which still differs by 0.11 tenth-meters from my determination.

With a view to explaining this difference, which is rather large considering the accuracy of both results, I undertook the following investigations with the large Bamberg spectrometer of the Astrophysical Observatory, and a Rowland plane grating.

1. The relative intensities of the two components of the double line in the arc spectrum of iron were determined and found to be in the ratio of 1 : 2 in the spectrum of the fourth order. Forming the weighted mean with these numbers we find the wave-length 4957.683 for the optical center of gravity of the pair.

2. As Keeler must have had to use a rather wide slit in making his measures, I tried by opening the slit to secure complete merging of the two components. The lines, however, never united into a single uniformly illuminated line, but the weaker component was always visible as a narrow border on the more refrangible side of the principal line. In spite of this I endeav-

ored to measure the wave-length of the pair through comparison with neighboring lines, and found $\lambda = 4957.685$. With the much lower dispersion of the spectrographs which I used, however, the pair of lines merged upon the photographic plate into a single sharp line. These two experiments completely confirm the value adopted by me for this line.

3. In view of the fact that Keeler did not use the arc spectrum but that of the spark, we might be led to suspect that the relative intensities of the two lines were quite different in the latter. This has proved to be the case. In a spark produced by a large induction coil and two Leyden jars the line 4957.480 was so weak as to be hardly visible by the side of the principal line. I estimated the ratio of intensities at 1 : 5. Hence I regard it as very probable that in the less brilliant spark which Keeler used for his comparison spectrum only the line λ 4957.785 was seen, and that the nebular line was referred to this. If we take the wave-length 4957.78 instead of 4957.63 for Keeler's comparison line, we find from his measures the value 4959.17 for the second nebular line. This agrees exactly with my determination.

The individual values of V determined with Spectrograph III show a very good agreement. It is, however, to be noted that in the case of *G. C.* 4390 the mean of the velocities derived from N_1 and N_2 must agree closely with that derived from $H\beta$ and $H\gamma$, since the wave-lengths of N_1 and N_2 have been obtained from the velocity determined originally from $H\beta$ and $H\gamma$. The results obtained with Spectograph I vary rather more widely. On a negative taken with this instrument a distance of 0.01 mm at the position of the nebular lines corresponds to a difference of wave-length of 0.46 tenth-meters, or a velocity of 28 km. As it has been found impossible on the most sensitive plates to measure a single line to a greater degree of accuracy than 0.002 mm, the individual velocity determinations made with this instrument will be uncertain by at least $\pm$ 5 km.

Using the wave-lengths I have derived for N_1 and N_2 we find from Table II the following determinations of velocity:

TABLE V.

Nebula	Plate	Line	$d\lambda$	V	Red. to Sun	V
G.C. 4390 West Edge (h 2000)	I 120	N_1	+ 0.32 t-m.	+19.2 km	−25.9 km	−6.7 km
Center............	I 123	N_1	+ 0.21	+12.6	−25.8	−13.2
		N_2	+ 0.17	+10.3	−25.8	−15.5
		$H\beta$	+ 0.18	+11.1	−25.8	−14.7
		$H\gamma$	+ 0.23	+15.9	−25.8	− 9.9
	III 390	N_1	+ 0.26	+15.6	−25.7	−10.1
		N_2	+ 0.29	+17.5	−25.7	− 8.2
		$H\beta$	+ 0.26	+16.0	−25.7	− 9.7
	III 392a	N_1	+ 0.27	+16.2	−25.6	− 9.4
		N_2	+ 0.25	+15.1	−25.6	−10.5
		$H\beta$	+ 0.26	+16.0	−25.6	− 9.6
	III 392b	N_1	+ 0.23	+13.8	−25.6	−11.8
		N_2	+ 0.26	+15.7	−25.6	− 9.9
		$H\beta$	+ 0.23	+14.2	−25.6	−11.4
G. C. 4373 Center .. (IV 37)	I 127	N_1	− 0.94	−56.3	− 0.1	−56.4
		N_2	− 0.91	−55.1	− 0.1	−55.2
		$H\beta$	− 0.95	−58.6	− 0.1	−58.7
		$H\gamma$	− 0.98	−67.7	− 0.1	−67.8
North Edge	III 389	N_1	− 1.15	−68.9	− 0.1	−69.0
N. G. C. 7027 Center (Webb)	I 144	N_1	+ 0.40	+24.2	−19.6	+ 4.6
		N_2	+ 0.41	+24.8	−19 6	+ 5.2

The results of Table V give the following mean values for the separate plates:

TABLE VI.

Nebula	Plate	V Hartmann	V Keeler
G. C. 4390 Edge (h 2000)	I 120	− 6.7 km	
Center	I 123	−13.3 (½)	
	III 390	− 9.3	
	III 392a	− 9.8	
	III 392b	−11.0	
	Mean	−10.5	−9.7 km
G. C. 4373 Center (IV 37)	I 127	−59.5 (½)	
Edge	III 389	−69.0	
	Mean	−65.8	−64.7
N. G. C. 7027 Center (Webb)	I 144	+ 4.9	+10.1

In view of what I have said above in regard to the limits of accuracy for the negatives made with Spectograph I, I should not wish to affirm that the slight difference of velocity between the edge and center of *G. C.* 4390 and *G. C.* 4373 is due to relative motion within the nebula. A more important fact, as it seems to me, is that on almost all of the photographs of the nebula *G. C.* 4390 the lines have a slight curvature as well as a slight inclination to the direction of the comparison lines. This makes the existence of relative motion within the nebula very probable. I have been unable to settle the question on account of the disappearance of the nebula in the western twilight, but I hope to be able to continue successfully the work outlined with the aid of a spectograph especially designed for the purpose. At the request of Director Vogel, who considers the smaller photographic refractor of the Observatory better adapted for the investigation of the spectra of extended nebulæ on account of its greater ratio of aperture to focal length, photography of the spectrum of the *Orion* nebula was begun by Dr. Eberhard in November of last year, and these plates have given more certain evidence as bearing on the existence of relative motion within the nebula.

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Minor Contributions and Notes

BANDS IN THE BUNSEN FLAME SPECTRUM OF SODIUM.

RECENTLY, while examining a Bunsen flame saturated with sodium chloride, using a pocket spectroscope which gives a very bright spectrum, the writer observed faint indications of several bands in the red. On introducing fresh salt into the flame, these bands flashed out quite distinctly. Three could be clearly seen, with some indications of a fourth further in the red. At times a fluted structure was suggested, with sharp edges toward the violet, but the details were too weak to admit of certainty on this point.

Several ordinary laboratory spectroscopes failed to show any trace of these bands, and it was believed that they must have been optical illusions, due to internal reflections; but finally, when using a spectrometer with larger lenses, of shorter focus, two of these bands could be faintly distinguished when a very intense sodium flame was used. Others beside the writer saw them. Settings were very uncertain, but the wave-lengths of these two bands were roughly determined to be about 6000 and 6100. The third band, seen only with the pocket spectroscope, must have been of wave-length about 6200.

It is, of course, possible that these bands were due to impurities; but they were given by six different kinds of sodium salts. Moreover, their positions are very nearly the same as those of three bands observed by Hartley[1] in the oxyhydrogen flame spectrum of sodium, of wave-lengths 6026, 6138, and 6233, so it seems probable that they were really due to sodium. So far as known to the writer, however, they have never before been seen in the Bunsen flame spectrum of sodium.

Eder and Valenta[2] have published a reproduction of their photograph of the Bunsen flame spectrum of sodium, taken with twenty-four hours' exposure. A very decided maximum of about wave-length

[1] *Phil. Trans.*, **185**, 177, 1899.

[2] *Denkschriften d. Wiener Ak.*, **60**, opp. p. 476, 1893.

6100 is very clearly shown on this plate, but no comment was made upon it by them.

This band is the brightest of those seen by the writer. It is interesting to note that Scheiner, in his *Astronomical Spectroscopy* (Frost's translation, p. 216), in referring to the outburst of sodium vapor in Comet 1882 I as it approached the Sun, states that the hydrocarbon bands previously seen were extinguished, while the red hydrocarbon band at $\lambda 6130$, previously unseen, was observed by Vogel.[1] Such an effect is obviously improbable. If a sodium band can appear in this neighborhood at comparatively low temperatures, it seems more reasonable to assume that this was a sodium band. It is also possible that the sodium vapor became fluorescent under the action of solar radiation, giving rise to the red band observed by Wiedemann and Schmidt[2] in the spectrum of fluorescing sodium vapor.

PERCIVAL LEWIS.

UNIVERSITY OF CALIFORNIA,
April 1902.

GRANT BY THE SMITHSONIAN INSTITUTION TO THE ASTROPHYSICAL JOURNAL.

THE readers of the ASTROPHYSICAL JOURNAL are doubtless aware that the expenses of a journal of this kind must greatly exceed the receipts, in spite of the fact that the number of subscribers is comparatively large and is rapidly increasing. Through the aid of a gift from Miss Catherine Wolfe Bruce in 1895, supplemented by a gift from Mrs. William Thaw, it has been possible to illustrate the JOURNAL freely, thus materially increasing its value. These special funds are now exhausted, after providing all the illustrations used in the JOURNAL since its foundation. It is therefore with special satisfaction that the editors announce the award by the Secretary of the Smithsonian Institution of a grant of two hundred dollars annually for the four years 1899–1903. This substantial assistance on the part of Secretary Langley, who has contributed so much to the development and the furtherance of astrophysical science, will be appreciated by all who are interested in the welfare of the JOURNAL.

[1] See also *A. N.*, **102**, No. 2437.

[2] ASTROPHYSICAL JOURNAL, **3**, 207, 1896.

MARIE-ALFRED CORNU.

In the untimely death on April 12 of Marie-Alfred Cornu, France loses one of her ablest physicists, the Ecole Polytechnique one of its most beloved teachers, and science one of its most eminent votaries.

M. Cornu has been an Associate Editor of this Journal since its foundation, and his loss will be keenly felt.

An appropriate sketch of his life will appear in an early number of the Journal.